Universal, deterministic, and exact protocol to reverse qubit-unitary and qubit-encoding isometry operations

Satoshi Yoshida (The University of Tokyo)
Akihito Soeda (National Institute of Informatics)
Mio Murao (The University of Tokyo)

arXiv:2209.02907

Outline

- General perspective

- Higher-order quantum transformations

- Result 1: Deterministic exact qubit-unitary inversion

SY, Akihito Soeda and Mio Murao, arXiv:2209.02907

- Result 2: Isometry inversion

SY, Akihito Soeda and Mio Murao, In preparation

- Future works

Outline

- General perspective

- Higher-order quantum transformations

- Result 1: Deterministic exact qubit-unitary inversion

SY, Akihito Soeda and Mio Murao, arXiv:2209.02907

This talk

- Result 2: Isometry inversion

SY, Akihito Soeda and Mio Murao, In preparation

- Future works

Outline

- General perspective

- Higher-order quantum transformations

- Result 1: Deterministic exact qubit-unitary inversion

SY, Akihito Soeda and Mio Murao, arXiv:2209.02907

- Result 2: Isometry inversion

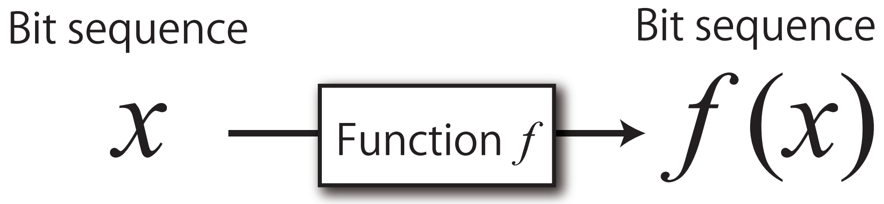
SY, Akihito Soeda and Mio Murao, In preparation

- Future works

Higher-order quantum transformation

- Classical information processing

- Function

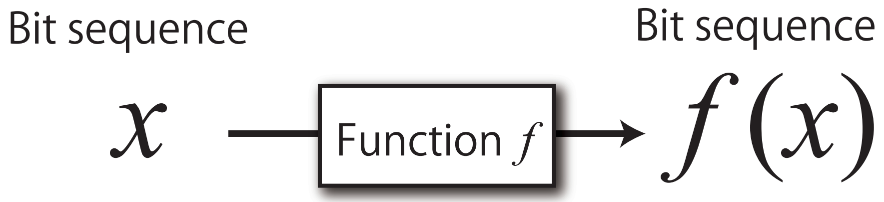


- Quantum information processing

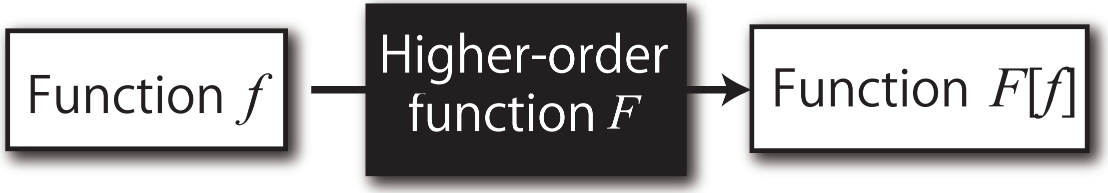
Higher-order quantum transformation

- Classical information processing

- Function



- Higher-order function

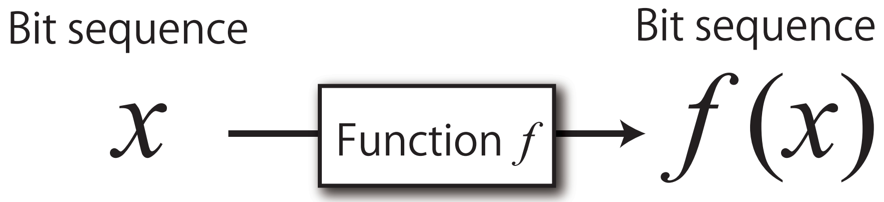


- Quantum information processing

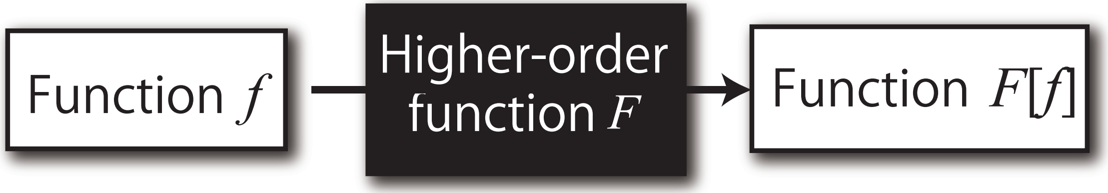
Higher-order quantum transformation

- Classical information processing

- Function

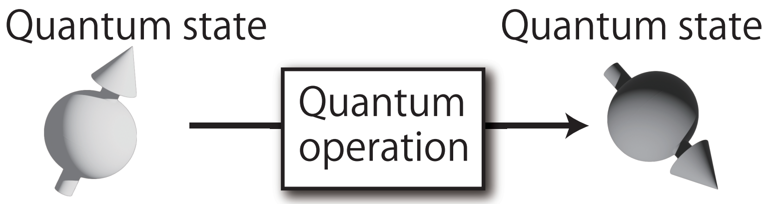


- Higher-order function

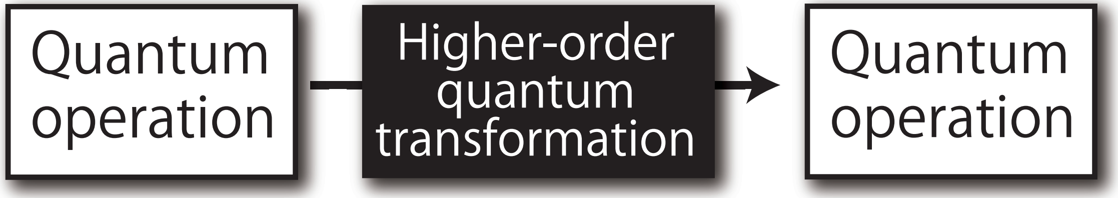


- Quantum information processing

- Quantum operation



- Higher-order quantum transformation

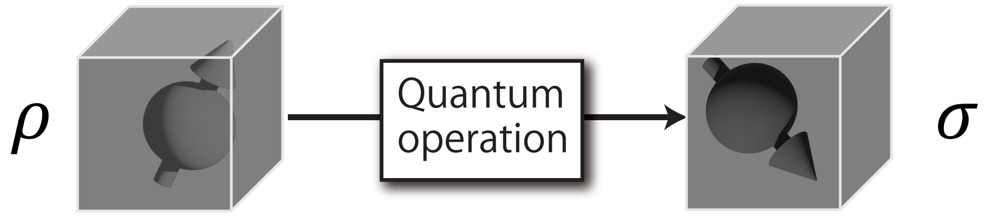


Universal transformation of quantum states

- Universal transformation of quantum states

Given: Unknown quantum state ρ

Task: Prepare a quantum state $\sigma = f(\rho)$



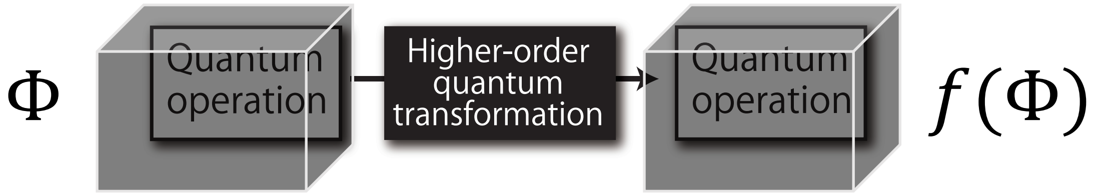
- Eg. State cloning

$$\rho \mapsto \rho \otimes \rho$$

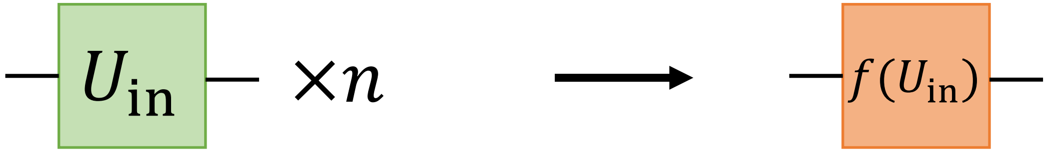
W. K. Wootters and W. H. Zurek, Nature 299, 802 (1982).

Universal transformation of quantum operations

- Given: Unknown quantum operation Φ
- Task: Implement a quantum operation $f(\Phi)$



- Eg. Universal transformation of unitary operation



$f(U) = U^{\otimes m}, U^*, U^{-1}, U^T, \mathrm{ctrl} - U, \ldots$

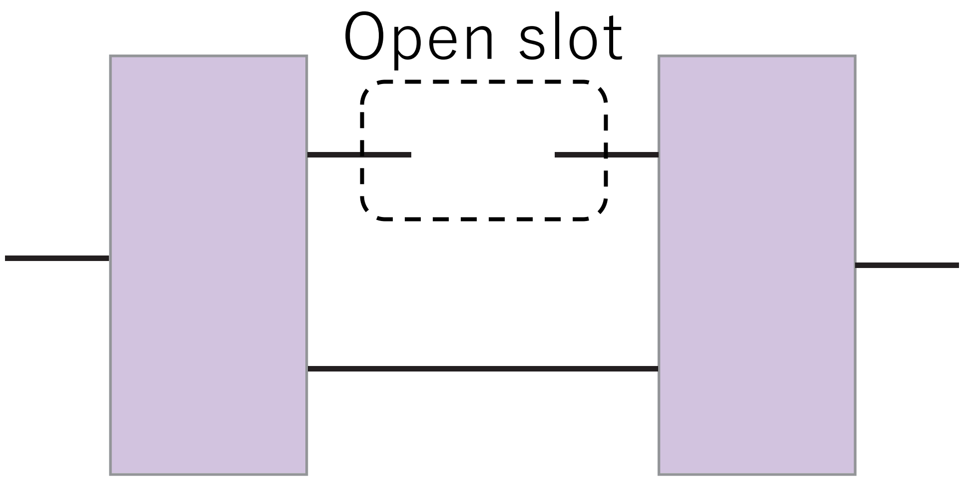
Unknown unitary

G. Chiribella et al. PRL 101, 180504 (2008). M. Quintino et al. PRL 123, 210502 (2019).
J. Miyazaki et al. PRR 1, 013007 (2019). D. Ebler et al. arXiv:2206.00107. Q. Dong at al. arXiv:1911.01645.

Quantum combs

• How to implement transformation of quantum operations?

→ Quantum circuit with open slot(s): Quantum comb

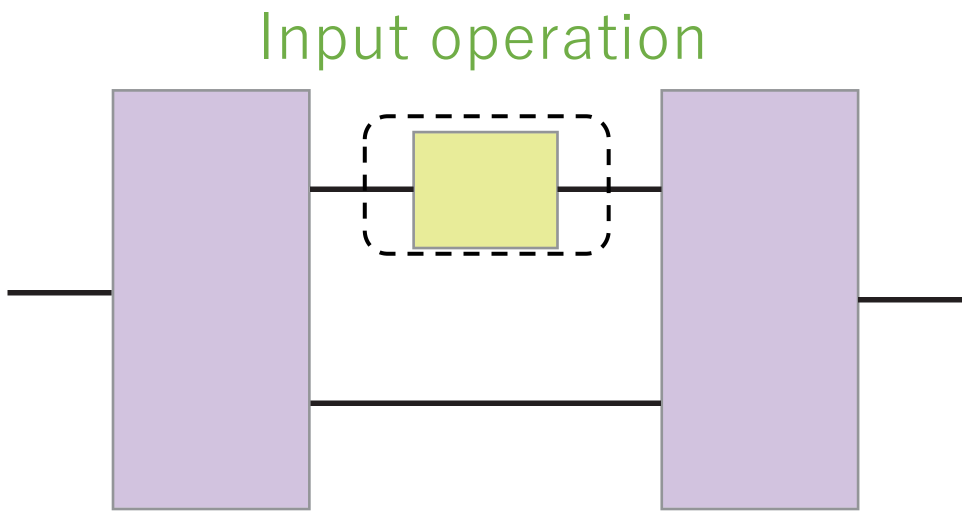


G. Chiribella et al. PRL 101, 060401 (2008).

Quantum combs

- How to implement transformation of quantum operations?

→ Quantum circuit with open slot(s): Quantum comb

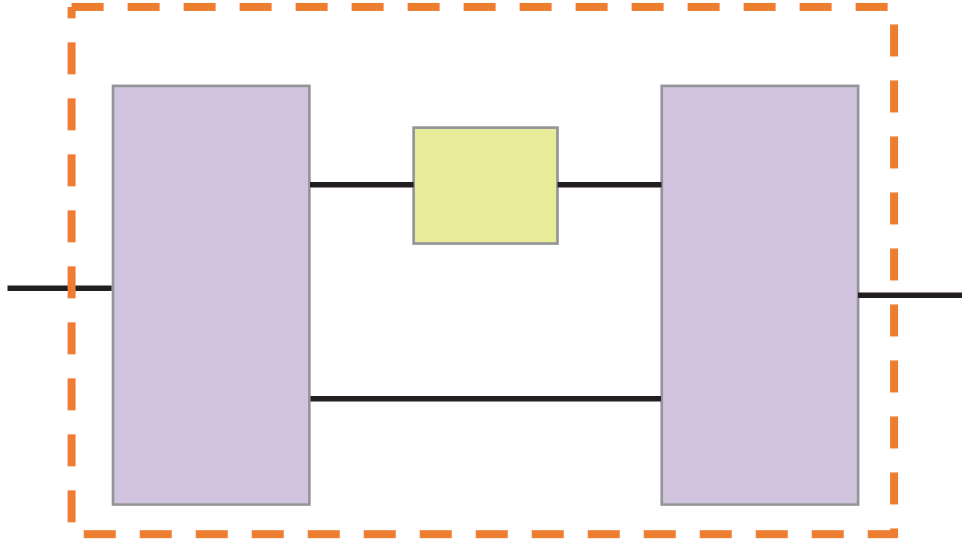


G. Chiribella et al. PRL 101, 060401 (2008).

Quantum combs

- How to implement transformation of quantum operations?

→ Quantum circuit with open slot(s): Quantum comb

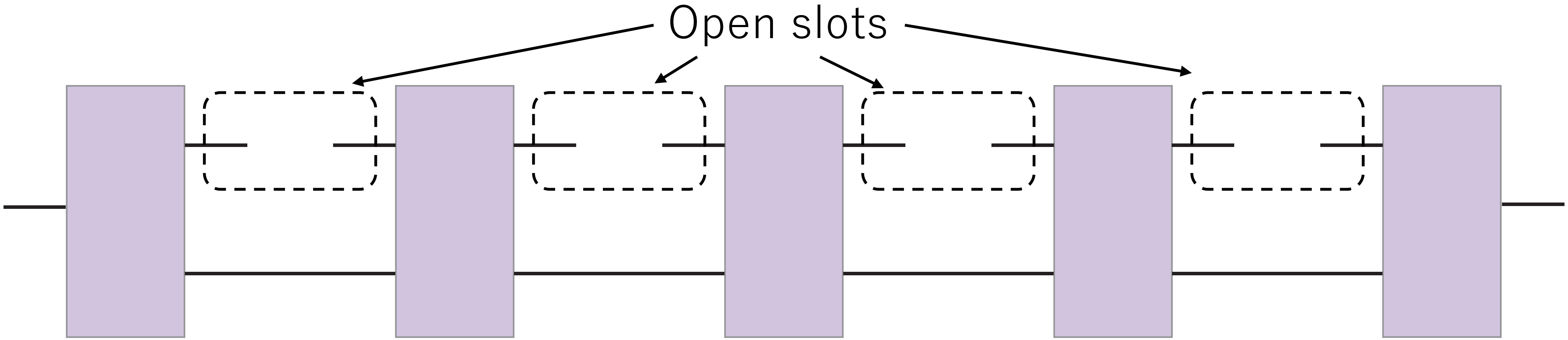


G. Chiribella et al. PRL 101, 060401 (2008).

Quantum combs

- How to implement transformation of quantum operations?

→ Quantum circuit with open slot(s): Quantum comb

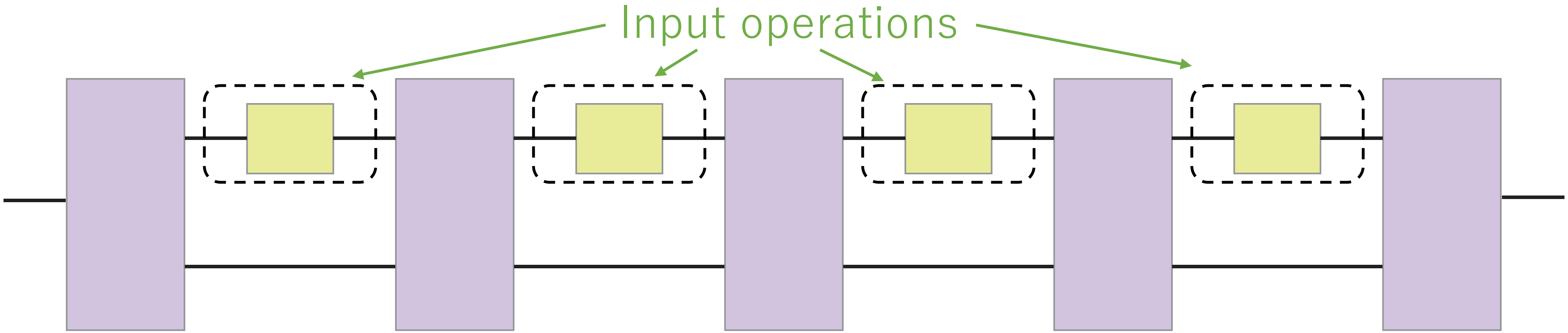


G. Chiribella et al. PRL 101, 060401 (2008).

Quantum combs

- How to implement transformation of quantum operations?

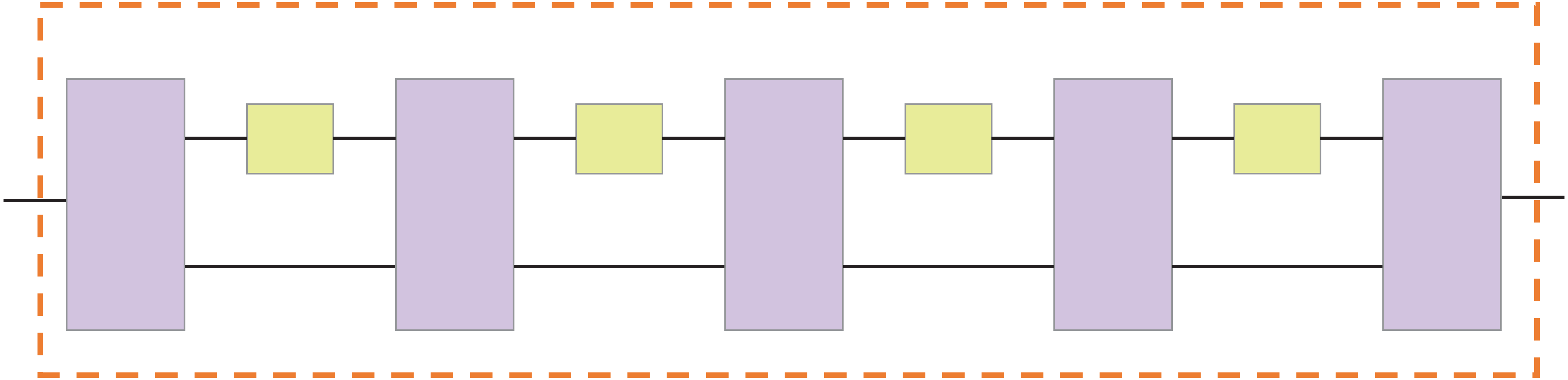
→ Quantum circuit with open slot(s): Quantum comb



G. Chiribella et al. PRL 101, 060401 (2008).

Quantum combs

• How to implement transformation of quantum operations?
→ Quantum circuit with open slot(s): Quantum comb



Output operation

G. Chiribella et al. PRL 101, 060401 (2008).

Outline

- General perspective
- Higher-order quantum transformations

- Result 1: Deterministic exact qubit-unitary inversion

SY, Akihito Soeda and Mio Murao, arXiv:2209.02907

- Result 2: Isometry inversion

SY, Akihito Soeda and Mio Murao, In preparation

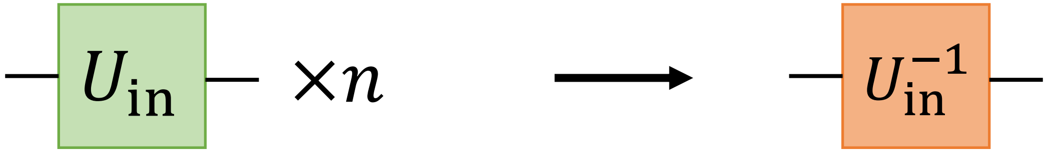
- Future works

Unitary inversion

- Task

Given: n calls of unknown unitary operation $U_{\rm in}$

Task: Implement the inverse operation $U_{\rm in}^{-1}$

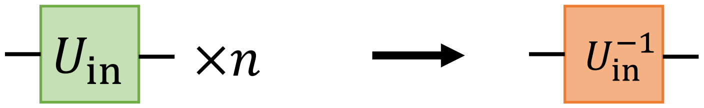


$\forall U_{\rm in} \in \mathrm{SU}(d)$

- Unitary inversion = simulation of "time inversion" $t \mapsto -t$

$$U_{\rm in} = e^{-iHt} \mapsto U_{\rm in}^{-1} = e^{iHt}$$

Unitary inversion



- Question:

-The fundamental limitation of unitary inversion?

- Previous work:

Go: probabilistic or non-exact algorithm

No-go: On the restricted class of protocols

M. Sedlak et al. PRL 122, 170502 (2019), M. Navascues, PRX 8, 031008 (2018), M. Quintino et al. PRL 123, 210502 (2019), M. Quintino et al. PRA 100, 062339 (2019). M. Quintino and D. Ebler Quantum 6, 679 (2022), I. S. Sardharwalla et al. arXiv: 1602.07963, D. Ebler et al. arXiv: 2206.00107, D. Trillo et al. Quantum 4, 374 (2020), D. Trillo et al. arXiv: 2205.00131, P. Schiansky et al. arXiv: 2205.01122.

Unitary inversion $U_{\text{in}} \times n \rightarrow U_{\text{in}}^{-1}$

- Question:

-The fundamental limitation of unitary inversion?

- Previous work:

Go: probabilistic or non-exact algorithm

No-go: On the restricted class of protocols

Open problem: Is it possible to implement deterministic and exact unitary inversion?

M. Sedlak et al. PRL 122, 170502 (2019), M. Navascues, PRX 8, 031008 (2018), M. Quintino et al. PRL 123, 210502 (2019), M. Quintino et al. PRA 100, 062339 (2019). M. Quintino and D. Ebler Quantum 6, 679 (2022), I. S. Sardharwalla et al. arXiv: 1602.07963, D. Ebler et al. arXiv: 2206.00107, D. Trillo et al. Quantum 4, 374 (2020), D. Trillo et al. arXiv: 2205.00131, P. Schiansky et al. arXiv: 2205.01122.

Unitary inversion

- Question:

-The fundamental limitation of unitary inversion?

- Previous work:

Go: probabilistic or non-exact algorithm

No-go: On the restricted class of p cols

Open problem: Is it pos ment deterministic and exact unitary invers

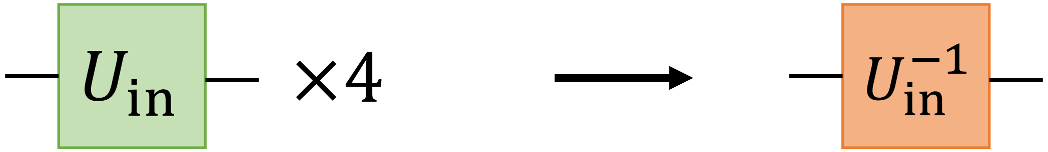
Answer positively for $d = 2$!

M. Sedlak et al. PRL 122, 170502 (s, PRX 8, 031008 (2018), M. Quintino et al. PRL 123, 210502 (2019), M. Quintino et al. PRA 100, 062339 (2019). M. Q . Ebler Quantum 6, 679 (2022), I. S. Sardharwalla et al. arXiv: 1602.07963, D. Ebler et al. arXiv: 2206.00107, D. Trillo et al. Q 4, 374 (2020), D. Trillo et al. arXiv: 2205.00131, P. Schiansky et al. arXiv: 2205.01122.

Unitary inversion

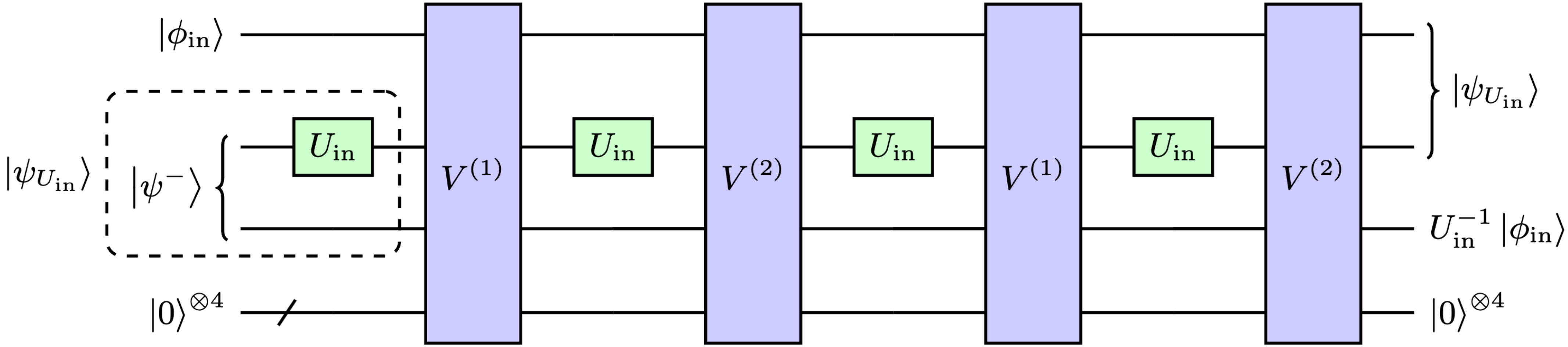
- Main result:

There exists a deterministic and exact qubit-unitary inversion protocol.



$\forall U_{\mathrm{in}} \in \mathrm{SU}(2)$

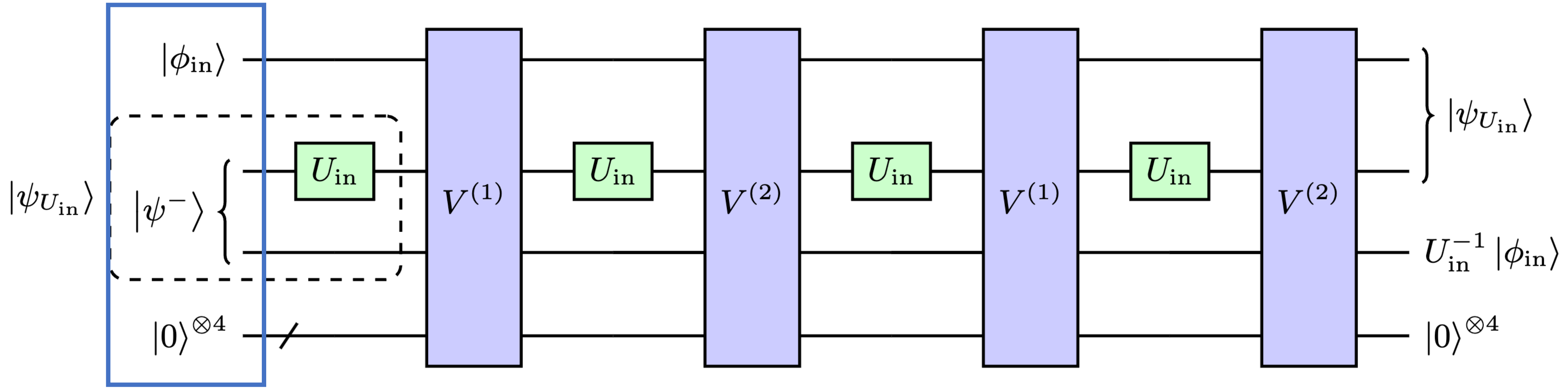
Qubit-unitary inversion protocol



- Task: Apply U_{in}^{-1} on the input quantum state $|\phi_{\mathrm{in}}\rangle$

1. Prepare $|\phi_{\mathrm{in}}\rangle$, $|\psi^-\rangle \coloneqq (|01\rangle - |10\rangle)/\sqrt{2}$ and $|0\rangle^{\otimes 4}$
2. Apply $U_{\mathrm{in}}\times 4$ and fixed unitary operations $V^{(1)}, V^{(2)}$ sequentially
3. We obtain $U_{\mathrm{in}}^{-1}|\phi_{\mathrm{in}}\rangle$, $|\psi_{U_{\mathrm{in}}}\rangle \coloneqq (U_{\mathrm{in}} \otimes I)|\psi^-\rangle$ and $|0\rangle^{\otimes 4}$

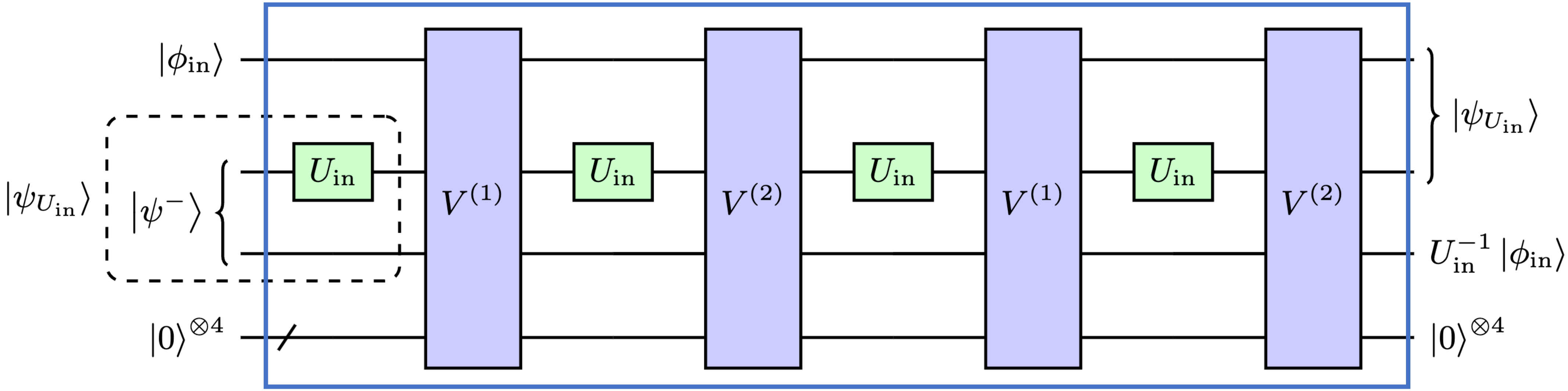
Qubit-unitary inversion protocol



- Task: Apply $U_{\rm in}^{-1}$ on the input quantum state $|\phi_{\rm in}\rangle$

1. Prepare $|\phi_{\rm in}\rangle$, $|\psi^-\rangle \coloneqq (|01\rangle - |10\rangle)/\sqrt{2}$ and $|0\rangle^{\otimes 4}$
2. Apply $U_{\rm in} \times 4$ and fixed unitary operations $V^{(1)}, V^{(2)}$ sequentially
3. We obtain $U_{\rm in}^{-1}|\phi_{\rm in}\rangle$, $|\psi_{U_{\rm in}}\rangle \coloneqq (U_{\rm in} \otimes I)|\psi^-\rangle$ and $|0\rangle^{\otimes 4}$

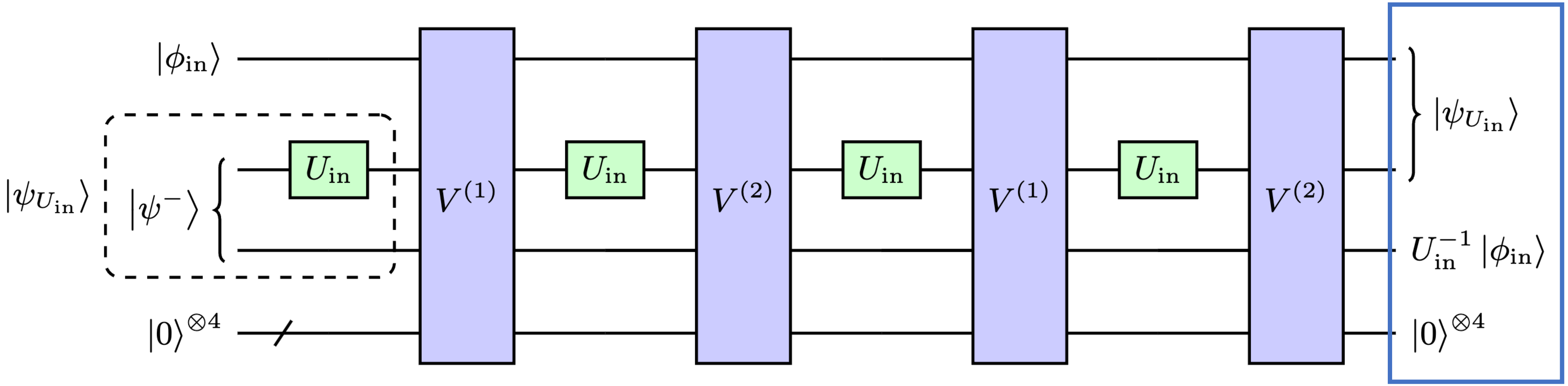
Qubit-unitary inversion protocol



- Task: Apply U_{in}^{-1} on the input quantum state $|\phi_{\text{in}}\rangle$

1. Prepare $|\phi_{\text{in}}\rangle$, $|\psi^-\rangle \coloneqq (|01\rangle - |10\rangle)/\sqrt{2}$ and $|0\rangle^{\otimes 4}$
2. Apply $U_{\text{in}}\times 4$ and fixed unitary operations $V^{(1)}, V^{(2)}$ sequentially
3. We obtain $U_{\text{in}}^{-1}|\phi_{\text{in}}\rangle$, $|\psi_{U_{\text{in}}}\rangle \coloneqq (U_{\text{in}} \otimes I)|\psi^-\rangle$ and $|0\rangle^{\otimes 4}$

Qubit-unitary inversion protocol



- Task: Apply U_{in}^{-1} on the input quantum state $|\phi_{\mathrm{in}}\rangle$

1. Prepare $|\phi_{\mathrm{in}}\rangle$, $|\psi^-\rangle \coloneqq (|01\rangle - |10\rangle)/\sqrt{2}$ and $|0\rangle^{\otimes 4}$
2. Apply $U_{\mathrm{in}} \times 4$ and fixed unitary operations $V^{(1)}, V^{(2)}$ sequentially
3. We obtain $U_{\mathrm{in}}^{-1}|\phi_{\mathrm{in}}\rangle$, $|\psi_{U_{\mathrm{in}}}\rangle \coloneqq (U_{\mathrm{in}} \otimes I)|\psi^-\rangle$ and $|0\rangle^{\otimes 4}$

Characteristics of this protocol

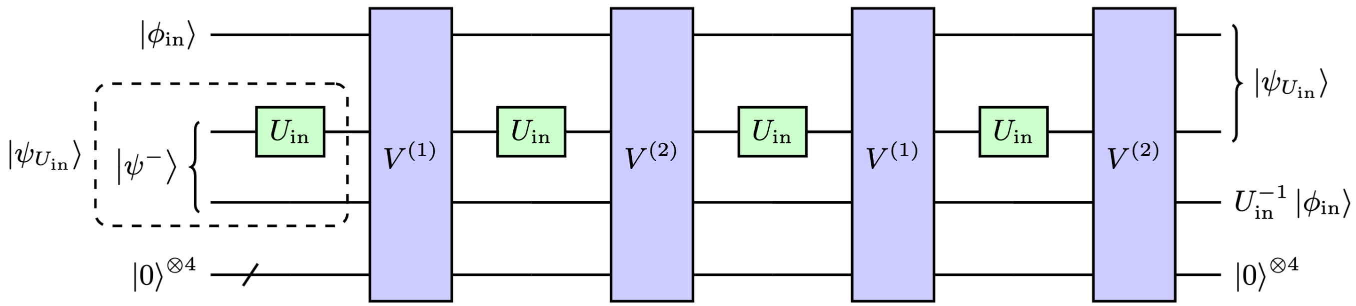
- Catalytic use of $|\psi_{U_{\text{in}}}\rangle$

$$|\phi_{\text{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{\text{in}}^{-1}|\phi_{\text{in}}\rangle \otimes |\psi_{U_{\text{in}}}\rangle \otimes |0\rangle^{\otimes 4}$$

- First call of U_{in} is used to prepare $|\psi_{U_{\text{in}}}\rangle := (U_{\text{in}} \otimes I)|\psi^-\rangle$

- The quantum state $|\psi_{U_{\text{in}}}\rangle$ is returned in the end

→ $|\psi_{U_{\text{in}}}\rangle$ can be reused to another run of unitary inversion!



Characteristics of this protocol

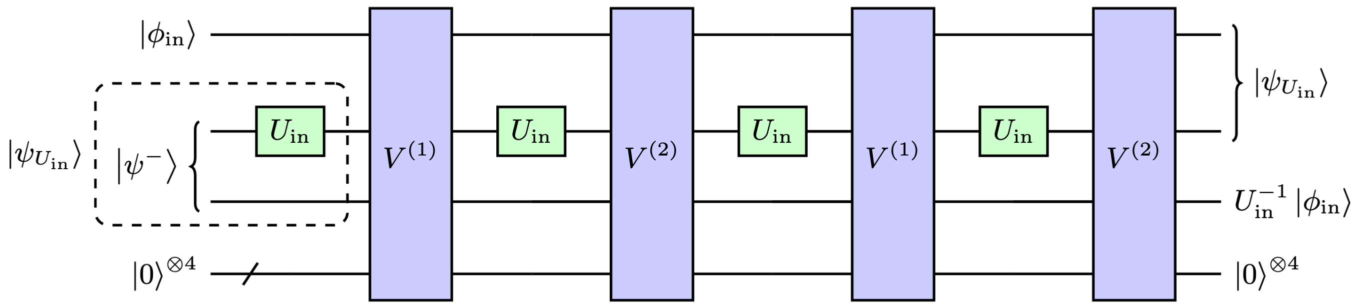
- Catalytic use of $|\psi_{U_{\text{in}}}\rangle$

$$|\phi_{\text{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{\text{in}}^{-1}|\phi_{\text{in}}\rangle \otimes |\psi_{U_{\text{in}}}\rangle \otimes |0\rangle^{\otimes 4}$$

\- First call of U_{in} is used to prepare $|\psi_{U_{\text{in}}}\rangle := (U_{\text{in}} \otimes I)|\psi^-\rangle$

\- The quantum state $|\psi_{U_{\text{in}}}\rangle$ is returned in the end

→ $|\psi_{U_{\text{in}}}\rangle$ can be reused to another run of unitary inversion!



Characteristics of this protocol

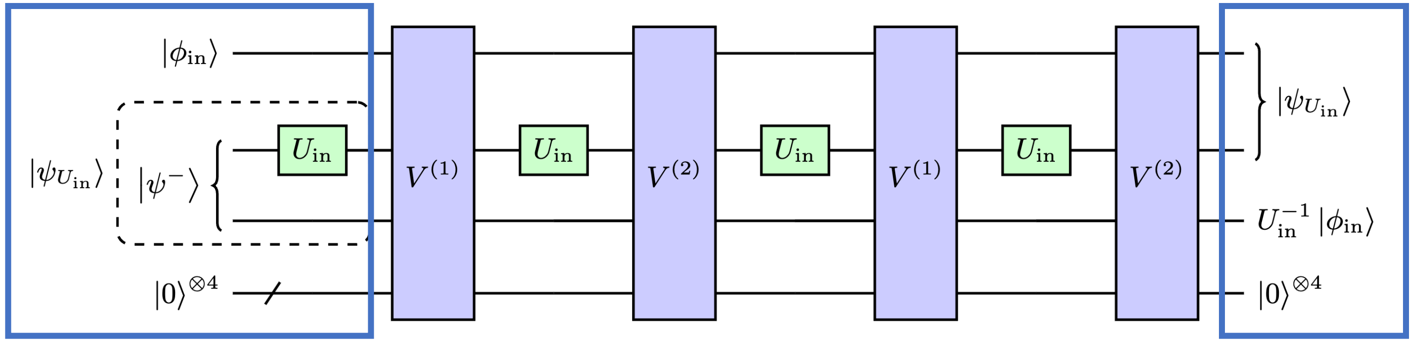
- Catalytic use of $|\psi_{U_{\mathrm{in}}}\rangle$

$$|\phi_{\mathrm{in}}\rangle \otimes |\psi_{U_{\mathrm{in}}}\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{\mathrm{in}}^{-1}|\phi_{\mathrm{in}}\rangle \otimes |\psi_{U_{\mathrm{in}}}\rangle \otimes |0\rangle^{\otimes 4}$$

\- First call of U_{in} is used to prepare $|\psi_{U_{\mathrm{in}}}\rangle := (U_{\mathrm{in}} \otimes I)|\psi^{-}\rangle$

\- The quantum state $|\psi_{U_{\mathrm{in}}}\rangle$ is returned in the end

→ $|\psi_{U_{\mathrm{in}}}\rangle$ can be reused to another run of unitary inversion!



Characteristics of this protocol

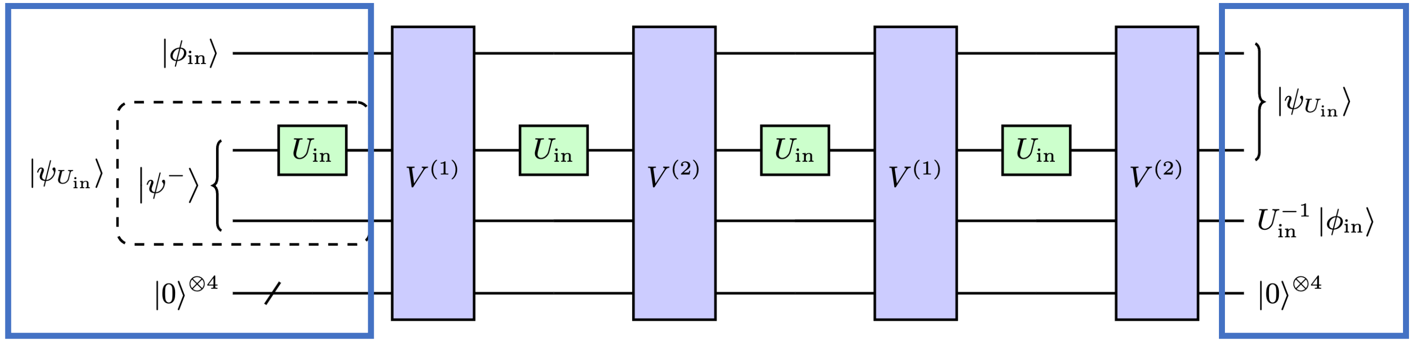
- Catalytic use of $|\psi_{U_{in}}\rangle$

$$|\phi_{in}\rangle \otimes |\psi_{U_{in}}\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{in}^{-1}|\phi_{in}\rangle \otimes |\psi_{U_{in}}\rangle \otimes |0\rangle^{\otimes 4}$$

- First call of U_{in} is used to prepare $|\psi_{U_{in}}\rangle := (U_{in} \otimes I)|\psi^-\rangle$

- The quantum state $|\psi_{U_{in}}\rangle$ is returned in the end

→ $|\psi_{U_{in}}\rangle$ can be reused to another run of unitary inversion!



Characteristics of this protocol

- Clean unitary inversion protocol

$$|\phi_{\text{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{\text{in}}^{-1}|\phi_{\text{in}}\rangle \otimes |\psi_{U_{\text{in}}}\rangle \otimes |0\rangle^{\otimes 4}$$

The output auxiliary state $|\psi_{U_{\text{in}}}\rangle$ stores the information of U_{in}

We can erase this information by applying an extra call of U_{in}:

$$(I \otimes U_{\text{in}})|\psi_{U_{\text{in}}}\rangle = U_{\text{in}}^{\otimes 2}|\psi^-\rangle = |\psi^-\rangle$$

Clean protocol: auxiliary output state does not depend on U_{in}

- Reversible computation: the auxiliary state is reset in the end

- Control unitary inversion: $\text{ctrl} - U_{\text{in}} \mapsto \text{ctrl} - U_{\text{in}}^{-1}$

Characteristics of this protocol

- Clean unitary inversion protocol

$$|\phi_{\text{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{\text{in}}^{-1}|\phi_{\text{in}}\rangle \otimes |\psi_{U_{\text{in}}}\rangle \otimes |0\rangle^{\otimes 4}$$

The output auxiliary state $|\psi_{U_{\text{in}}}\rangle$ stores the information of U_{in}

We can erase this information by applying an extra call of U_{in}:

$$(I \otimes U_{\text{in}})|\psi_{U_{\text{in}}}\rangle = U_{\text{in}}^{\otimes 2}|\psi^-\rangle = |\psi^-\rangle$$

Clean protocol: auxiliary output state does not depend on U_{in}

- Reversible computation: the auxiliary state is reset in the end
- Control unitary inversion: $\text{ctrl} - U_{\text{in}} \mapsto \text{ctrl} - U_{\text{in}}^{-1}$

Characteristics of this protocol

- Clean unitary inversion protocol

$$|\phi_{\text{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{\text{in}}^{-1}|\phi_{\text{in}}\rangle \otimes |\psi_{U_{\text{in}}}\rangle \otimes |0\rangle^{\otimes 4}$$

The output auxiliary state $|\psi_{U_{\text{in}}}\rangle$ stores the information of U_{in}

We can erase this information by applying an extra call of U_{in}:

$$(I \otimes U_{\text{in}})|\psi_{U_{\text{in}}}\rangle = U_{\text{in}}^{\otimes 2}|\psi^-\rangle = |\psi^-\rangle$$

Clean protocol: auxiliary output state does not depend on U_{in}

- Reversible computation: the auxiliary state is reset in the end

- Control unitary inversion: $\text{ctrl} - U_{\text{in}} \mapsto \text{ctrl} - U_{\text{in}}^{-1}$

Characteristics of this protocol

- Clean unitary inversion protocol

$$|\phi_{\mathrm{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{\mathrm{in}}^{-1}|\phi_{\mathrm{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4}$$

The output auxiliary state $|\psi_{U_{\mathrm{in}}}\rangle$ stores the information of U_{in}

We can erase this information by applying an extra call of U_{in}:

$$(I \otimes U_{\mathrm{in}})|\psi_{U_{\mathrm{in}}}\rangle = U_{\mathrm{in}}^{\otimes 2}|\psi^-\rangle = |\psi^-\rangle$$

Clean protocol: auxiliary output state does not depend on U_{in}

- Reversible computation: the auxiliary state is reset in the end

- Control unitary inversion: $\mathrm{ctrl} - U_{\mathrm{in}} \mapsto \mathrm{ctrl} - U_{\mathrm{in}}^{-1}$

Characteristics of this protocol

- Clean unitary inversion protocol

$$|\phi_{\mathrm{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{\mathrm{in}}^{-1}|\phi_{\mathrm{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4}$$

The output auxiliary state $|\psi_{U_{\mathrm{in}}}\rangle$ stores the information of U_{in}

We can erase this information by applying an extra call of U_{in}:

$$(I \otimes U_{\mathrm{in}})|\psi_{U_{\mathrm{in}}}\rangle = U_{\mathrm{in}}^{\otimes 2}|\psi^-\rangle = |\psi^-\rangle$$

Clean protocol: auxiliary output state does not depend on U_{in}

- Reversible computation: the auxiliary state is reset in the end

- Control unitary inversion: $\mathrm{ctrl} - U_{\mathrm{in}} \mapsto \mathrm{ctrl} - U_{\mathrm{in}}^{-1}$

Characteristics of this protocol

- Clean unitary inversion protocol

$$|\phi_{\mathrm{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{\mathrm{in}}^{-1}|\phi_{\mathrm{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4}$$

The output auxiliary state $|\psi_{U_{\mathrm{in}}}\rangle$ stores the information of U_{in}

We can erase this information by applying an extra call of U_{in}:

$$(I \otimes U_{\mathrm{in}})|\psi_{U_{\mathrm{in}}}\rangle = U_{\mathrm{in}}^{\otimes 2}|\psi^-\rangle = |\psi^-\rangle$$

Clean protocol: auxiliary output state does not depend on U_{in}

- Reversible computation: the auxiliary state is reset in the end

- Control unitary inversion: $\mathrm{ctrl} - U_{\mathrm{in}} \mapsto \mathrm{ctrl} - U_{\mathrm{in}}^{-1}$

How to find this protocol?
: Numerical search + symmetry

- Deterministic exact unitary inversion exists

$\Leftrightarrow$ The solution of the following SDP is 1:

$$\max \mathrm{Tr}(C\Omega)$$
$$\mathrm{s.t.}\ \ C \text{ is a quantum comb}$$

M. Quintino and D. Ebler, Quantum 6, 679 (2022)

- Size of C : $d^{2(n+1)} \times d^{2(n+1)}$ → Hard to calculate

d: dimension, n: number of calls

- A certain symmetry of the performance operator Ω

$$[\Omega, V^{\otimes n+1} \otimes W^{\otimes n+1}] = 0 \quad \forall V, W \in \mathrm{SU}(d)$$

→ We can impose an additional constraint:

$$[C, V^{\otimes n+1} \otimes W^{\otimes n+1}] = 0 \quad \forall V, W \in \mathrm{SU}(d)$$

How to find this protocol? : Numerical search + symmetry

- Deterministic exact unitary inversion exists

⟺ The solution of the following SDP is 1:

$$\max \mathrm{Tr}(C\Omega)$$
$$\text{s.t. } C \text{ is a quantum comb}$$

M. Quintino and D. Ebler, Quantum 6, 679 (2022)

- Size of C : $d^{2(n+1)} \times d^{2(n+1)}$ → Hard to calculate

d: dimension, n: number of calls

- A certain symmetry of the performance operator Ω

$$[\Omega, V^{\otimes n+1} \otimes W^{\otimes n+1}] = 0 \quad \forall V, W \in \mathrm{SU}(d)$$

→ We can impose an additional constraint:

$$[C, V^{\otimes n+1} \otimes W^{\otimes n+1}] = 0 \quad \forall V, W \in \mathrm{SU}(d)$$

How to find this protocol?
: Numerical search + symmetry

- Deterministic exact unitary inversion exists

$\Leftrightarrow$ The solution of the following SDP is 1:

$$\max \mathrm{Tr}(C\Omega)$$
$$\mathrm{s.t.}\ \ C \text{ is a quantum comb}$$

M. Quintino and D. Ebler, Quantum 6, 679 (2022)

- Size of C : $d^{2(n+1)} \times d^{2(n+1)}$ → Hard to calculate

d: dimension, n: number of calls

- A certain symmetry of the performance operator Ω

$$\left[\Omega, V^{\otimes n+1} \otimes W^{\otimes n+1}\right] = 0 \quad \forall V, W \in \mathrm{SU}(d)$$

→ We can impose an additional constraint:

$$\left[C, V^{\otimes n+1} \otimes W^{\otimes n+1}\right] = 0 \quad \forall V, W \in \mathrm{SU}(d)$$

How to find this protocol?
: Numerical search + symmetry

- Deterministic exact unitary inversion exists

$\Leftrightarrow$ The solution of the following SDP is 1:

$$\max \mathrm{Tr}(C\Omega)$$
$$\mathrm{s.t.}\ \ C \text{ is a quantum comb}$$

M. Quintino and D. Ebler, Quantum 6, 679 (2022)

- Size of C : $d^{2(n+1)} \times d^{2(n+1)}$ → Hard to calculate

d: dimension, n: number of calls

- A certain symmetry of the performance operator Ω

$$[\Omega, V^{\otimes n+1} \otimes W^{\otimes n+1}] = 0 \quad \forall V, W \in \mathrm{SU}(d)$$

→ We can impose an additional constraint:

$$[C, V^{\otimes n+1} \otimes W^{\otimes n+1}] = 0 \quad \forall V, W \in \mathrm{SU}(d)$$

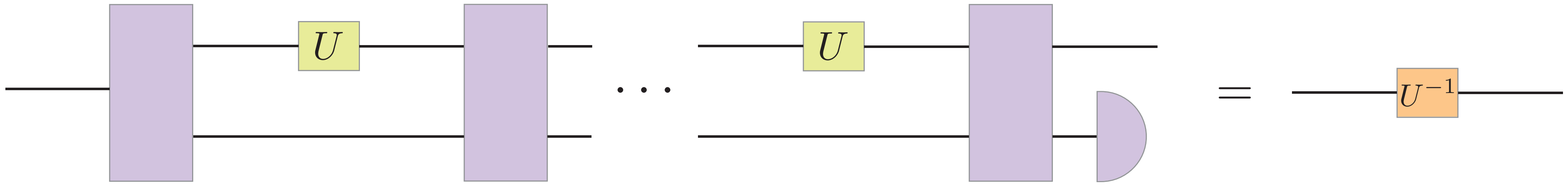
Reduction of SDP using $\mathbf{SU}(d)\times\mathbf{SU}(d)$ symmetry

M. Quintino et al. PRA 100, 062339 (2019)

- Symmetry in unitary inversion protocol

① $U \mapsto VUW$ for $V, W \in \mathrm{SU}(d)$

② Insert V and W to the whole circuit



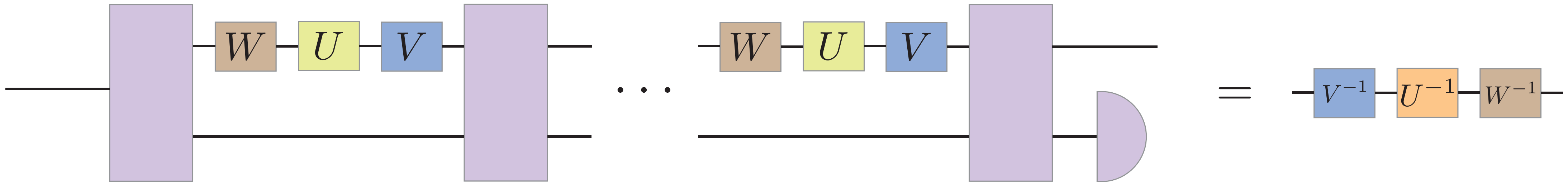
Reduction of SDP using $\mathbf{SU}(d)\times\mathbf{SU}(d)$ symmetry

M. Quintino et al. PRA 100, 062339 (2019)

- Symmetry in unitary inversion protocol

① $U \mapsto VUW$ for $V, W \in \mathrm{SU}(d)$

② Insert V and W to the whole circuit



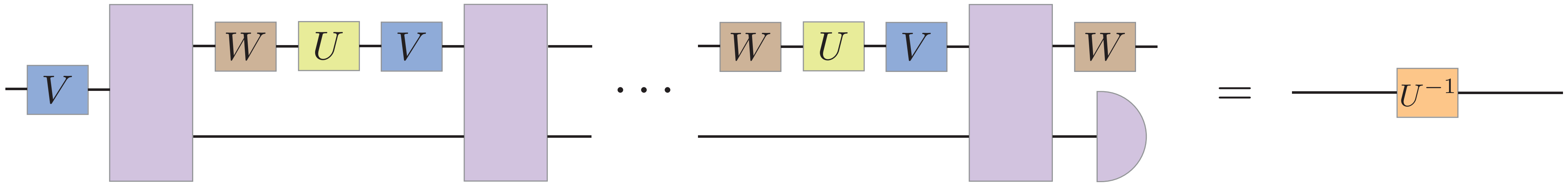
Reduction of SDP using $\mathbf{SU}(d)\times\mathbf{SU}(d)$ symmetry

M. Quintino et al. PRA 100, 062339 (2019)

- Symmetry in unitary inversion protocol

① $U \mapsto VUW$ for $V, W \in \mathrm{SU}(d)$

② Insert V and W to the whole circuit



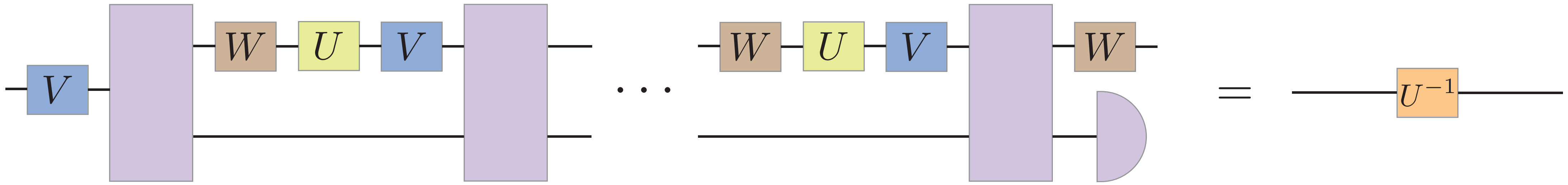
Reduction of SDP using $\mathbf{SU}(d)\times\mathbf{SU}(d)$ symmetry

M. Quintino et al. PRA 100, 062339 (2019)

- Symmetry in unitary inversion protocol

① $U \mapsto VUW$ for $V, W \in \mathrm{SU}(d)$

② Insert V and W to the whole circuit



→ Corresponds to the $\mathrm{SU}(d)\times\mathrm{SU}(d)$ symmetry of Ω

$$[\Omega, V^{\otimes n+1} \otimes W^{\otimes n+1}] = 0 \quad \forall V, W \in \mathrm{SU}(d)$$

Reduction of SDP using $\mathbf{SU}(d)\times\mathbf{SU}(d)$ symmetry

- Schur-Weyl duality：

An operator X commutes with $V^{\otimes n+1}$ for all $V \in \mathrm{SU}(d)$

$\Leftrightarrow X$ is a linear combination of permutation operators P_σ, where

$$P_\sigma|i_1,\cdots,i_{n+1}\rangle := |i_{\sigma^{-1}(1)},\cdots,i_{\sigma^{-1}(n+1)}\rangle$$

- $[C, V^{\otimes n+1} \otimes W^{\otimes n+1}] = 0 \quad \forall V, W \in \mathrm{SU}(d)$

$\Longrightarrow C = \sum_{\sigma,\tau\in\mathfrak{S}_{n+1}} c_{\sigma\tau} P_\sigma \otimes P_\tau$, where $c_{\sigma\tau} \in \mathbb{C}$

Numerical calculation of the SDP

- Numerical calculation of the SDP

Previous work: $d = 2, n \leq 3$ or $d = 3, n \leq 2$

This work : arbitrary $d, n \leq 5$

	$d = 2$	$d = 3$	$d = 4$	$\cdots$
$n = 2$	Previous			
$n = 3$				
$n = 4$		This work		
$n = 5$				

- $d = 2, n = 4$ → Deterministic exact qubit-unitary inversion

- We construct a qubit-unitary inversion circuit from the numerical result using [A. Bisio et al. PRA 83, 022325 (2011)]

Note: Similar technique is used in [D. Grinko and M. Ozols, arXiv:2207.05713]

Outline

- General perspective of my research topic
- Higher-order quantum transformations

- Result 1: Deterministic exact qubit-unitary inversion

SY, Akihito Soeda and Mio Murao, arXiv:2209.02907

- Result 2: Isometry inversion

SY, Akihito Soeda and Mio Murao, In preparation

- Future works

Isometry inversion

SY, Akihito Soeda and Mio Murao, Quantum 7, 957 (2023)

- Isometry inversion:

Given: n uses of an unknown isometry operation V

Task: Implement its inverse operation $\tilde{\mathcal{V}}_{\mathrm{inv}}$ s.t. $\tilde{\mathcal{V}}_{\mathrm{inv}} \circ \tilde{\mathcal{V}} = \mathrm{id}$

d: input dimension of isometry
D: output dimension of isometry

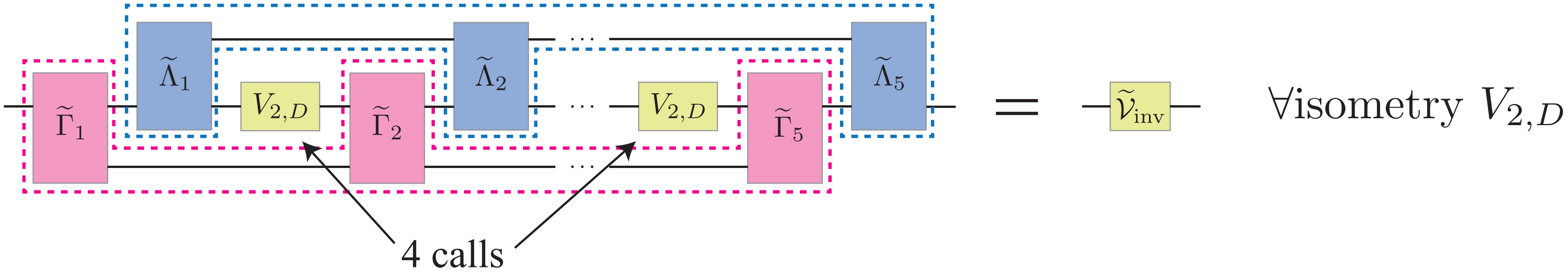
Encoder

Decoder

Isometry inversion

- Result:

There exists a deterministic exact protocol to reverse any qubit-encoding $(d = 2)$ isometry operations.

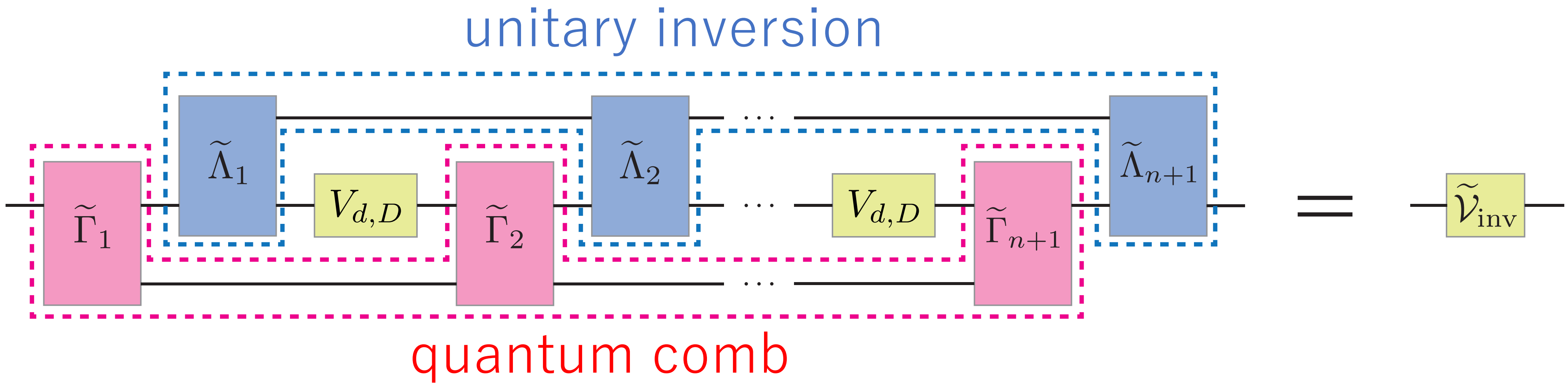


SY, Akihito Soeda and Mio Murao, In preparation

Proof sketch

- Key idea

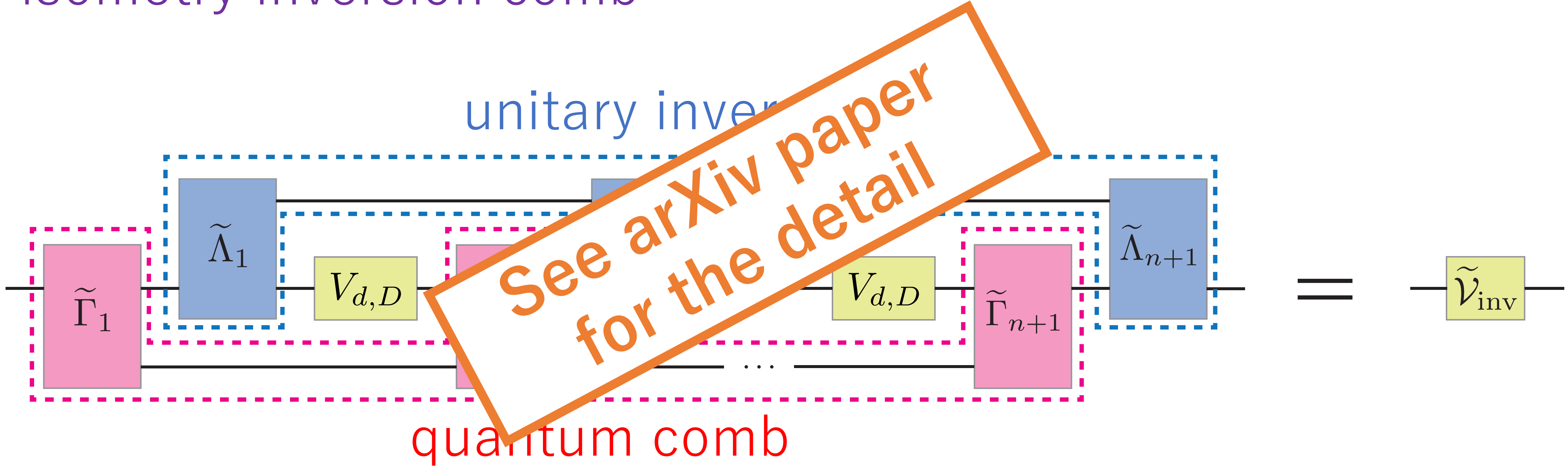
Use a quantum comb to transform unitary inversion comb into isometry inversion comb



Proof sketch

- Key idea

Use a quantum comb to transform unitary inversion comb into isometry inversion comb



Outline

- General perspective
- Higher-order quantum transformations

- Result 1: Deterministic exact qubit-unitary inversion

SY, Akihito Soeda and Mio Murao, arXiv:2209.02907

- Result 2: Isometry inversion

SY, Akihito Soeda and Mio Murao, arXiv:2110.00258 + In preparation

- Future works

Future works

- Extension of deterministic exact unitary inversion for higher-dimensions $d > 2$

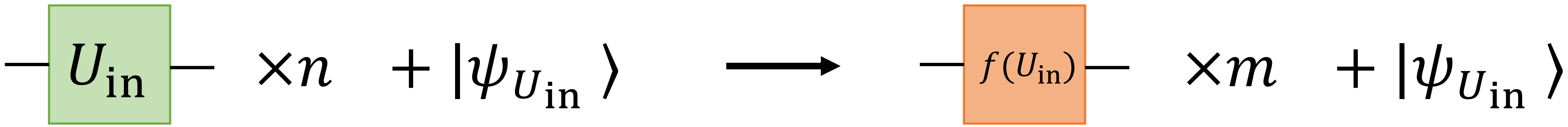
$$\forall U_{\mathrm{in}} \in \mathrm{SU}(d)$$

- Is it possible for arbitrary d?

- If so, how many times we need to call the input operation?

Future works

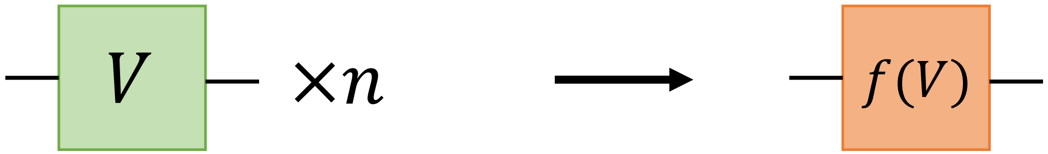
- Catalytic higher-order quantum transformations



- How catalyst helps in other tasks?
- What kind of catalyst states are useful?

Future works

- More universal transformations of isometry operations



- Isometry $\supset$ [Unitary $(D = d)$ $\cup$ Pure state $(d = 1)$]

→Unified understanding of universal transformations of unitary operations and quantum states?

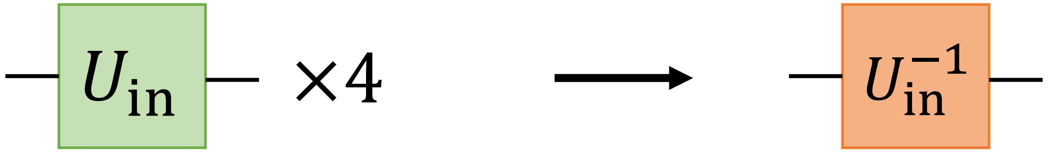
Eg. State cloning vs. Unitary cloning

G. Chiribella et al. Phys. Rev. Lett. 101, 180504 (2008).

Summary

arXiv:2209.02907

- Deterministic exact qubit-unitary inversion



$\forall U_{\mathrm{in}} \in SU(2)$

$$|\phi_{\mathrm{in}}\rangle \otimes |\psi^-\rangle \otimes |0\rangle^{\otimes 4} \mapsto U_{\mathrm{in}}^{-1}|\phi_{\mathrm{in}}\rangle \otimes |\psi_{U_{\mathrm{in}}}\rangle \otimes |0\rangle^{\otimes 4}$$

- Catalytic use of $|\psi_{U_{\mathrm{in}}}\rangle \coloneqq (U_{\mathrm{in}} \otimes I)|\psi^-\rangle$
- Clean-version protocol