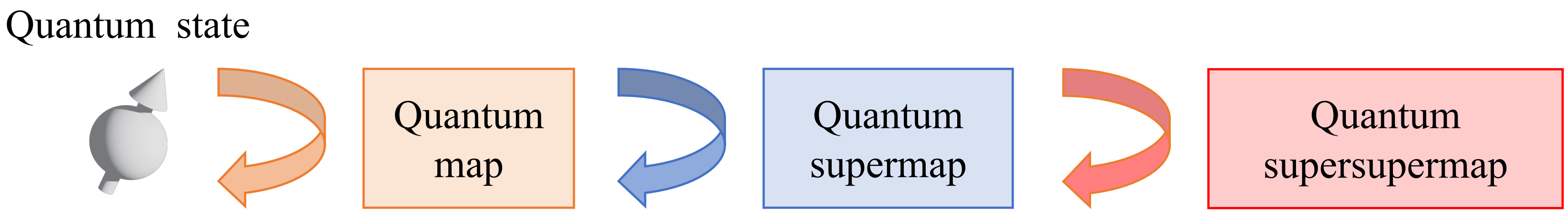
Optimal protocols for universal adjointation of isometry operations

Satoshi Yoshida (UTokyo)
Joint work with Akihito Soeda (NII), Mio Murao (UTokyo)

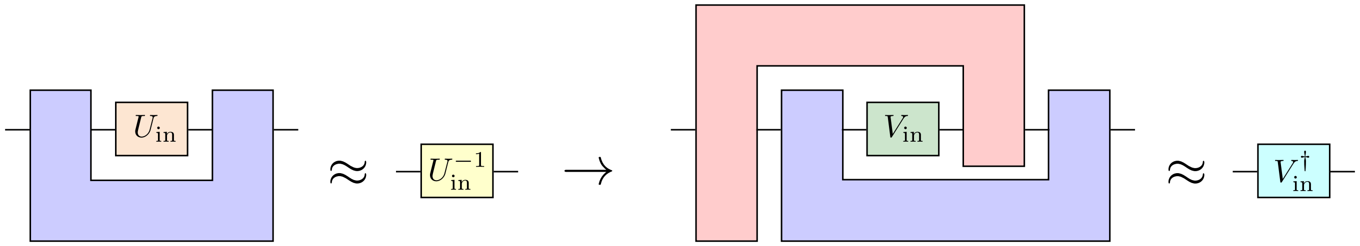
arXiv:2401.10137

Outline

- Higher-order quantum computation



- Result: Optimal construction of isometry adjointation



- Future works

Outline

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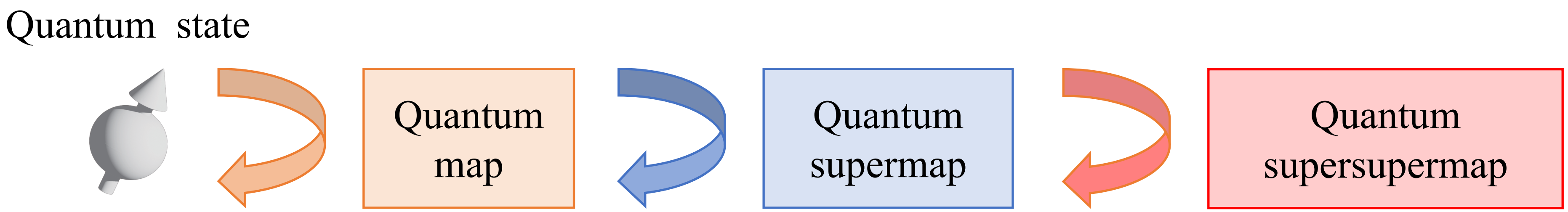

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- Future works

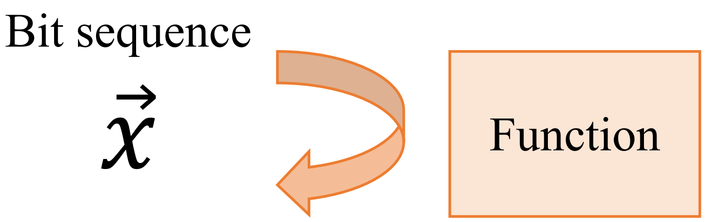
Higher-order quantum computation

Bit sequence

$\vec{x}$

A. Bisio and P. Perinotti, Proc. R. Soc. A 475, 20180706 (2019).

Higher-order quantum computation



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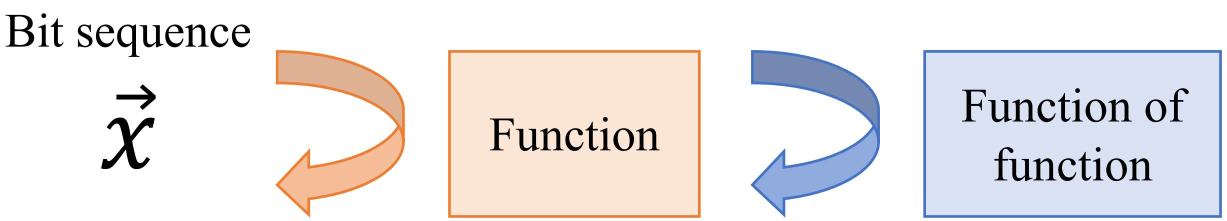

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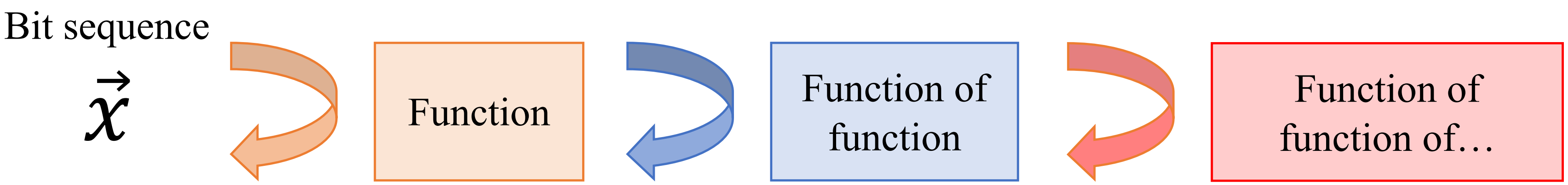

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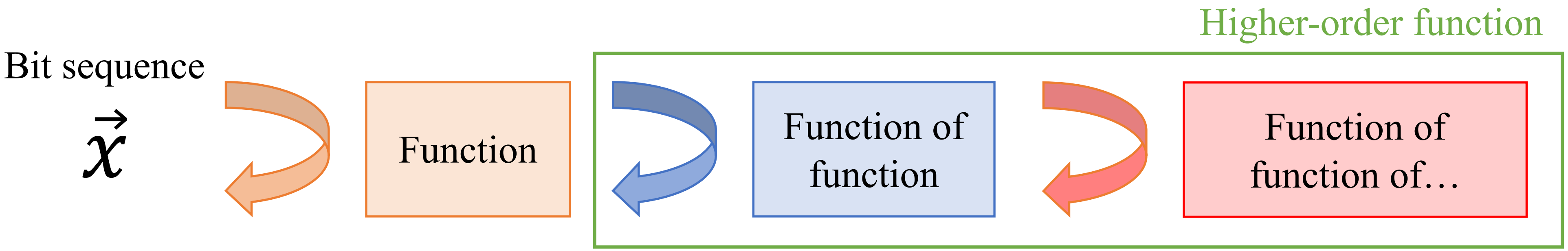

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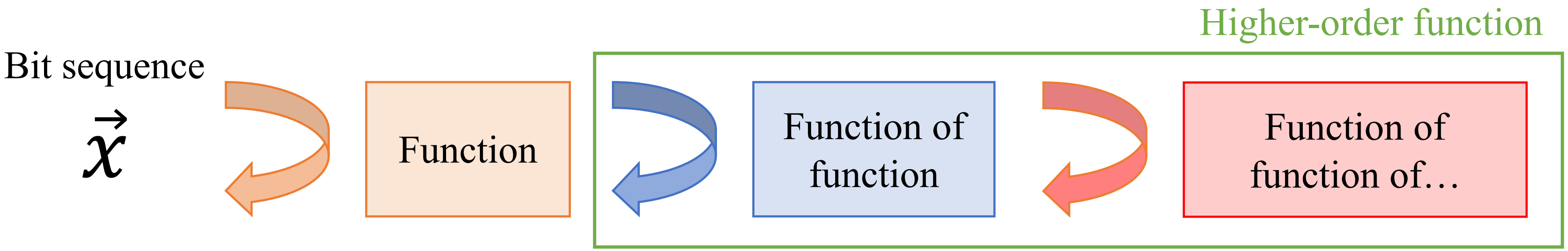
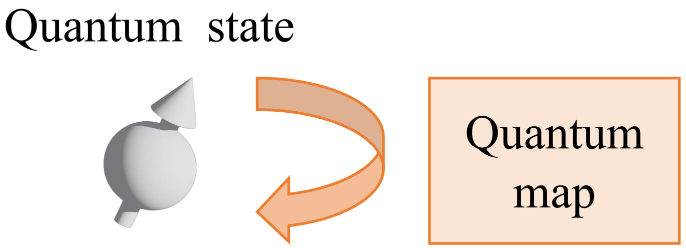


- Quantum version



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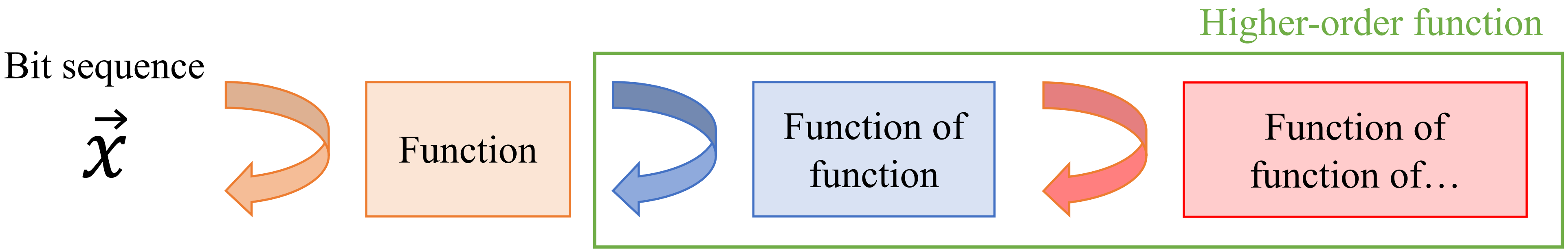

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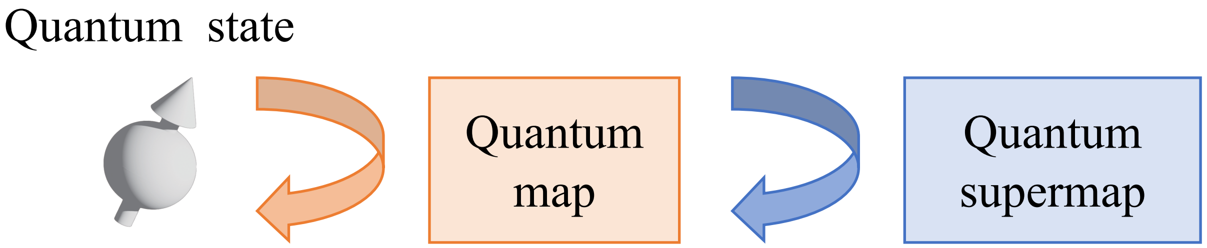

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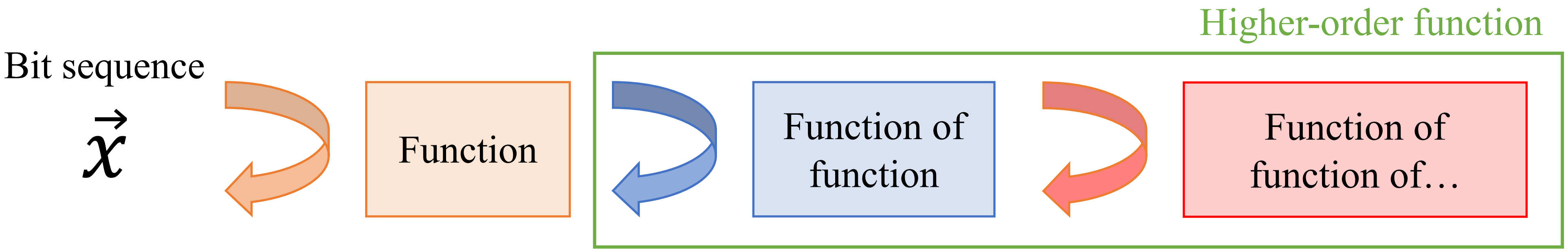

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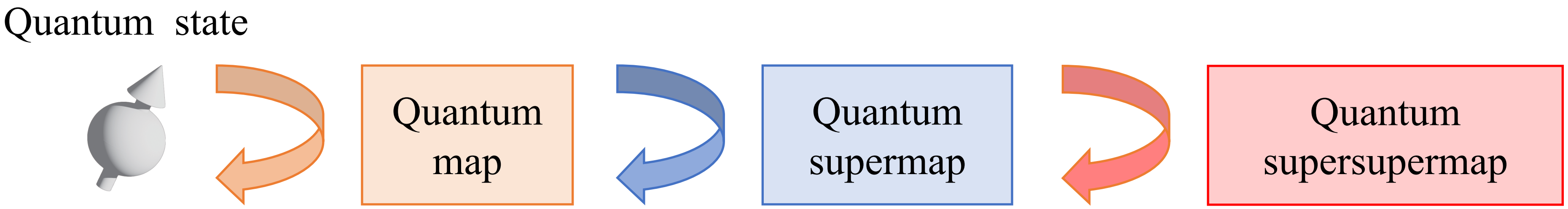

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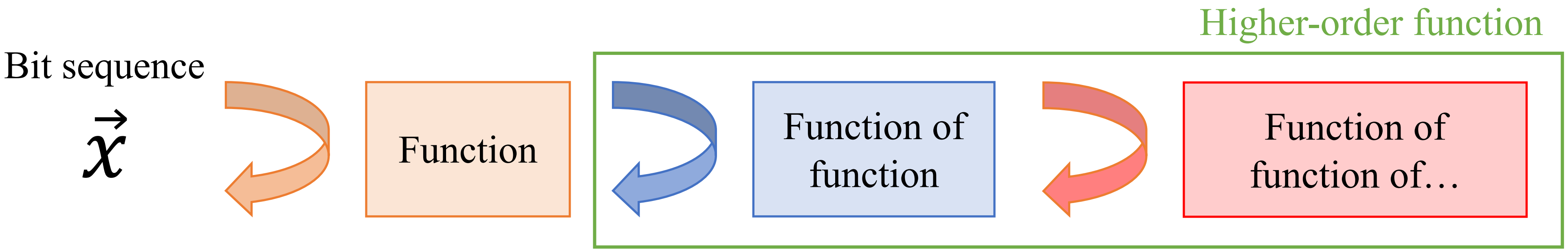

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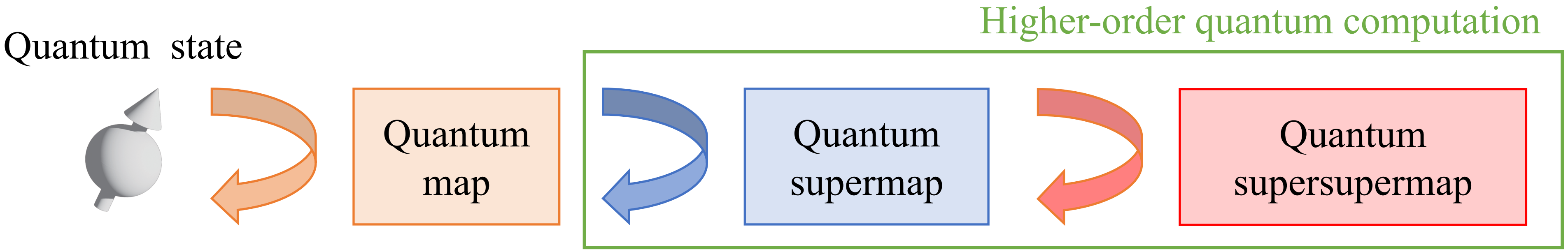

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Circuit implementation of supermaps

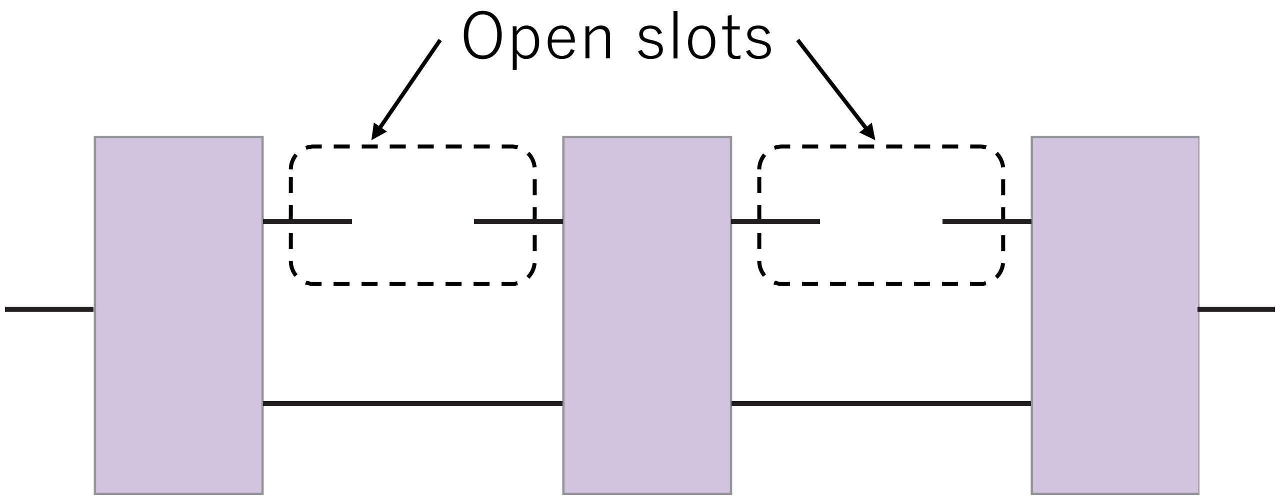
- How to implement quantum supermaps?

G. Chiribella et al. PRL 101, 060401 (2008).

Circuit implementation of supermaps

- How to implement quantum supermaps?

→ Quantum circuit with open slots: Quantum comb



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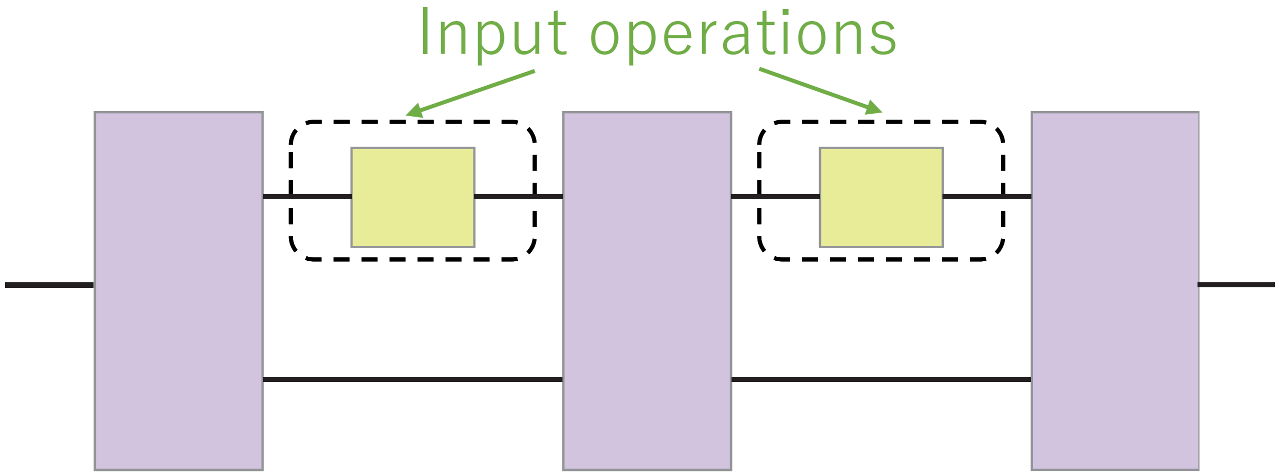

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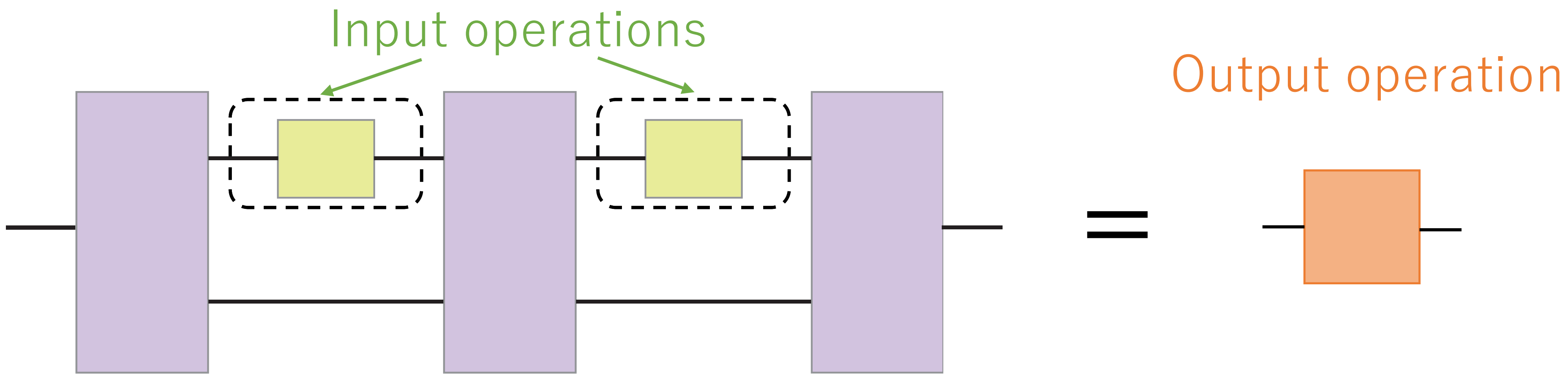

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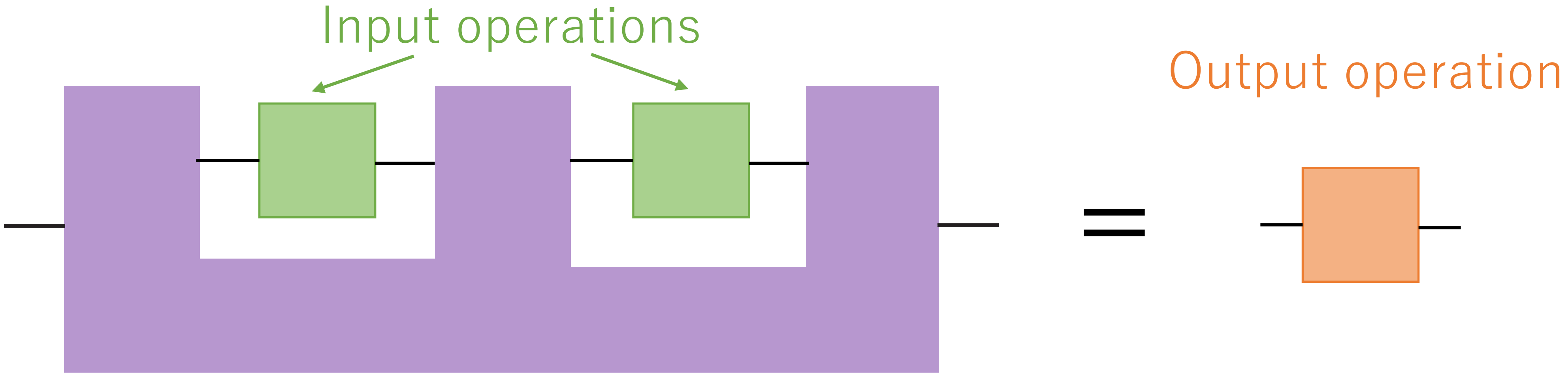

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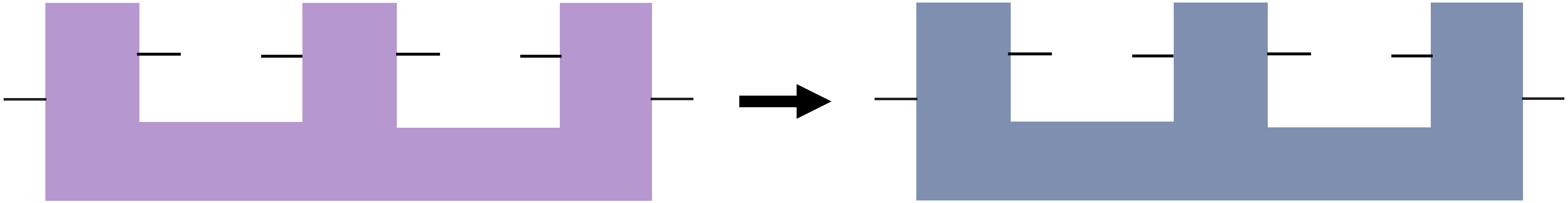

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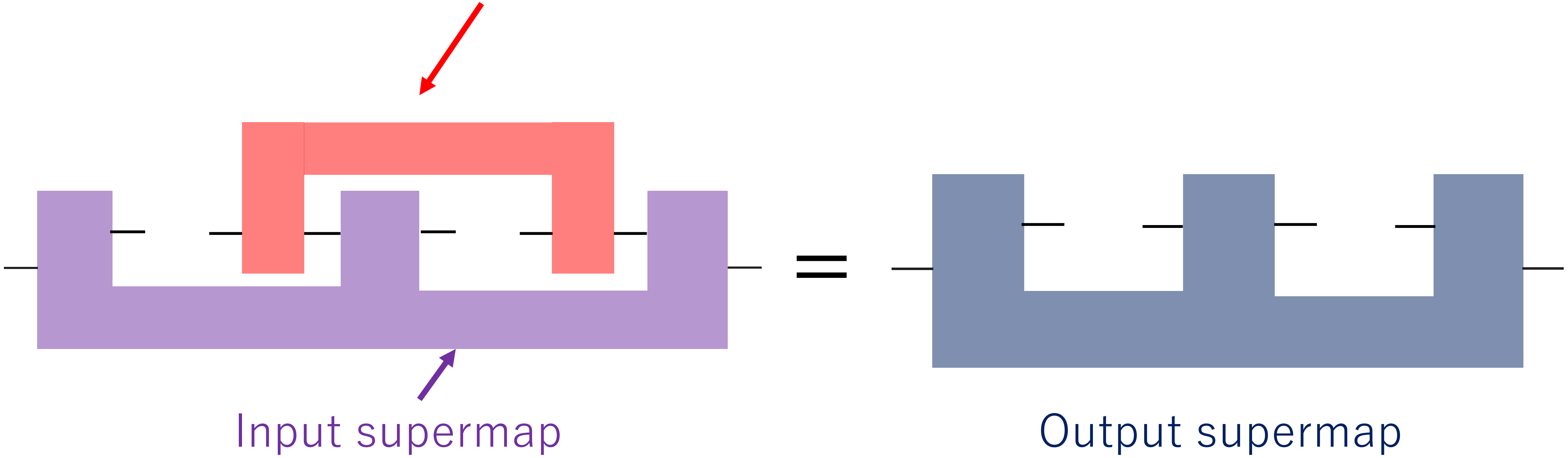
Circuit implementation of supersupermaps

- How to implement quantum supersupermaps?

→ Insert another quantum comb



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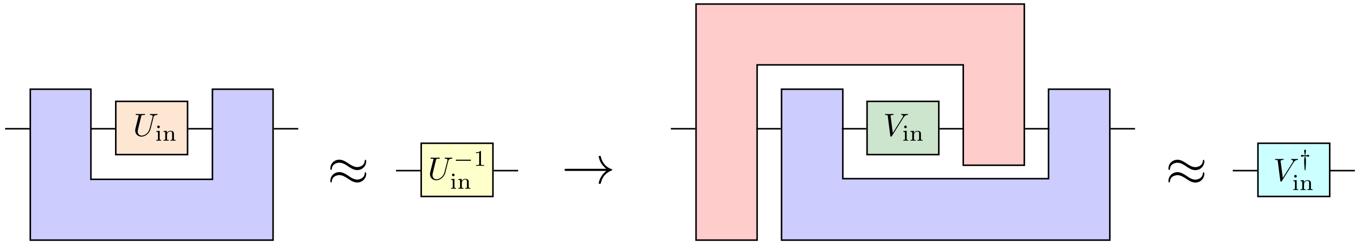
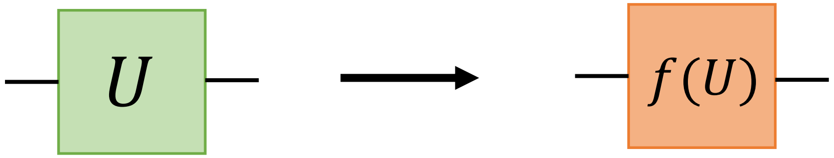


- Future works

Motivation

- Typical task of higher-order quantum computation

Given



Unknown operation

- What kind of f is implementable? $f(U) = U^{\otimes m}, U^*, U^{-1}, U^T, \mathrm{ctrl} - U, \ldots$

- Extension to non-unitary operation?

- Relationship between f and $g = F[f]$?

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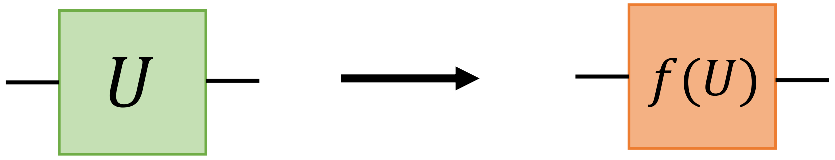

Unknown operation

- What kind of f is implementable? $f(U) = U^{\otimes m}, U^*, U^{-1}, U^T, \mathrm{ctrl} - U, \ldots$

- Extension to non-unitary operation? → Isometry operation

- Relationship between f and $g = F[f]$? → Supersupermap

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Unitary inversion

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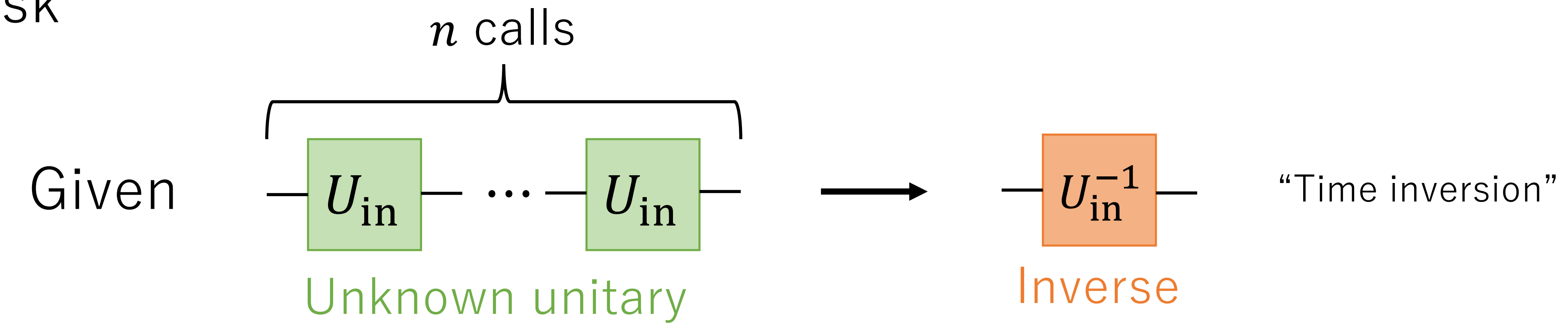

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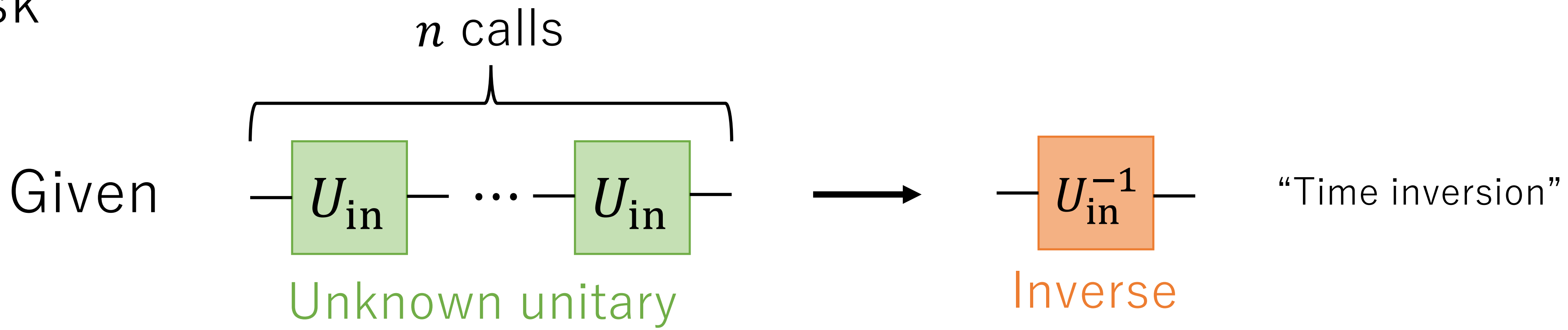


- Two possible extensions to isometry

Isometry inversion

$$V_{\mathrm{in}} \mapsto V_{\mathrm{in}}^{-1}$$

Isometry adjointation

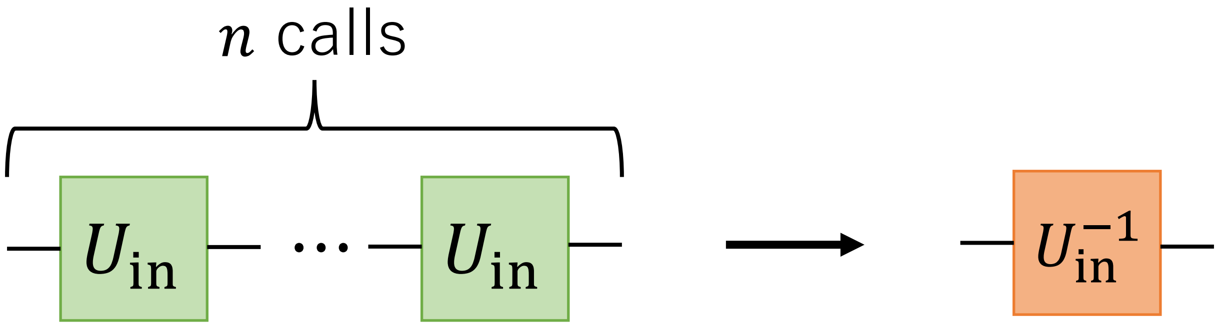
$$V_{\mathrm{in}} \mapsto V_{\mathrm{in}}^{\dagger}$$

SY, A. Soeda, M. Murao, Quantum 7, 957 (2023).

Unitary inversion

- Task

Given



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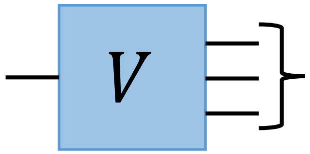
This work

SY, A. Soeda, M. Murao, Quantum 7, 957 (2023).

Adjoint of isometry operation

- Isometry operations

Eg. $\alpha|0\rangle + \beta|1\rangle$

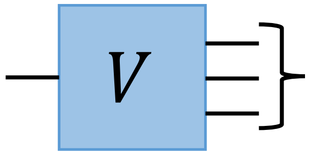


$\alpha|000\rangle + \beta|111\rangle$

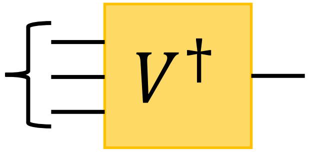
Encoder

Adjoint of isometry operation

- Isometry operations

Eg. $\alpha|0\rangle + \beta|1\rangle$  $\alpha|000\rangle + \beta|111\rangle$

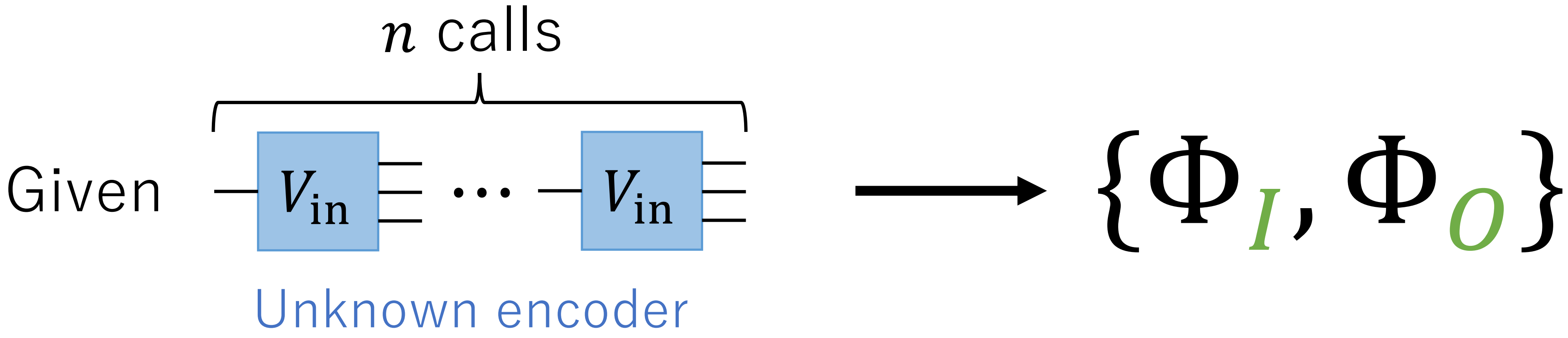
Encoder

$\alpha|000\rangle + \beta|111\rangle + \gamma|001\rangle$  $\alpha|0\rangle + \beta|1\rangle$

Subspace check & decode

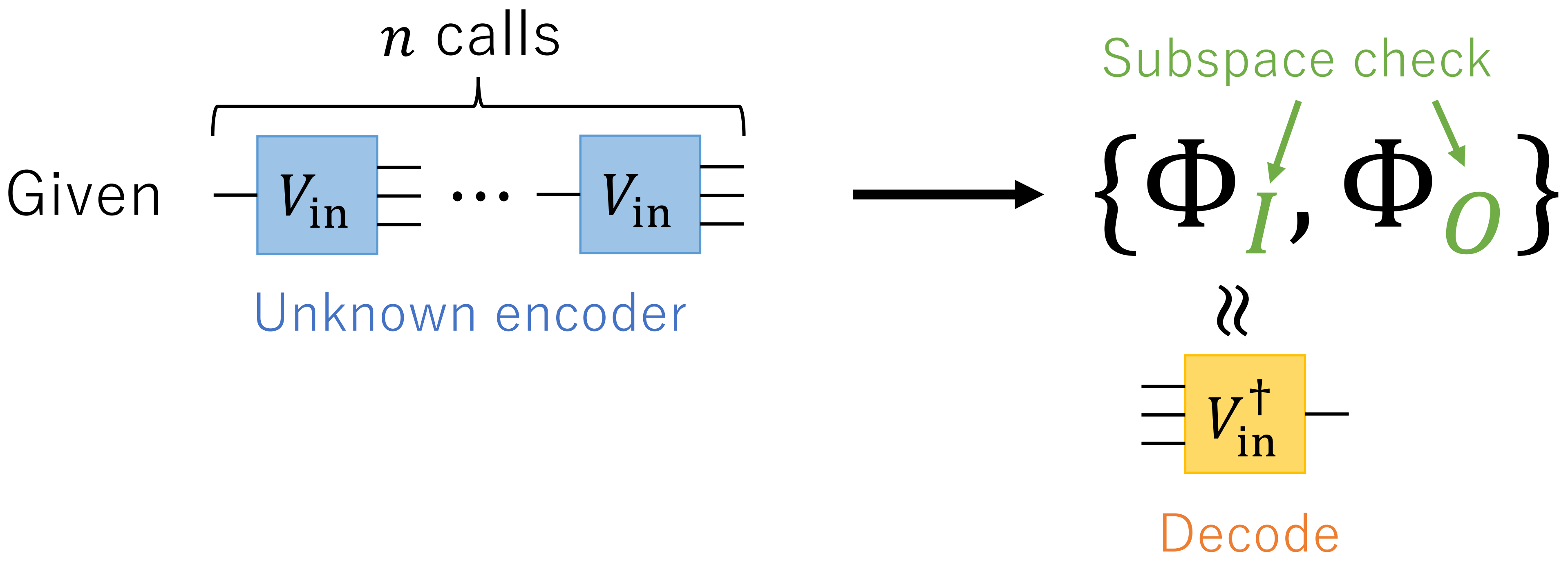
Isometry adjointation

- Task:



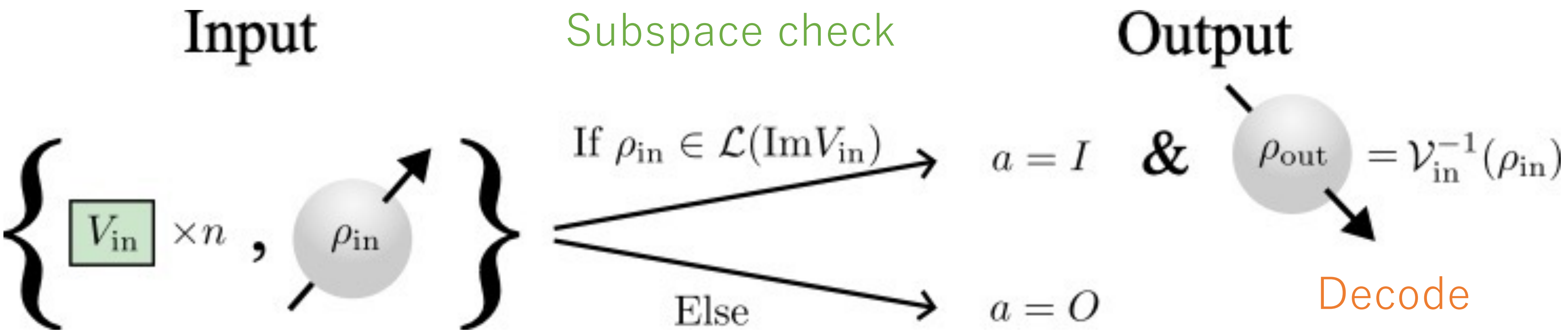
Isometry adjointation

- Task:



Isometry adjointation

- In other words, the task is:

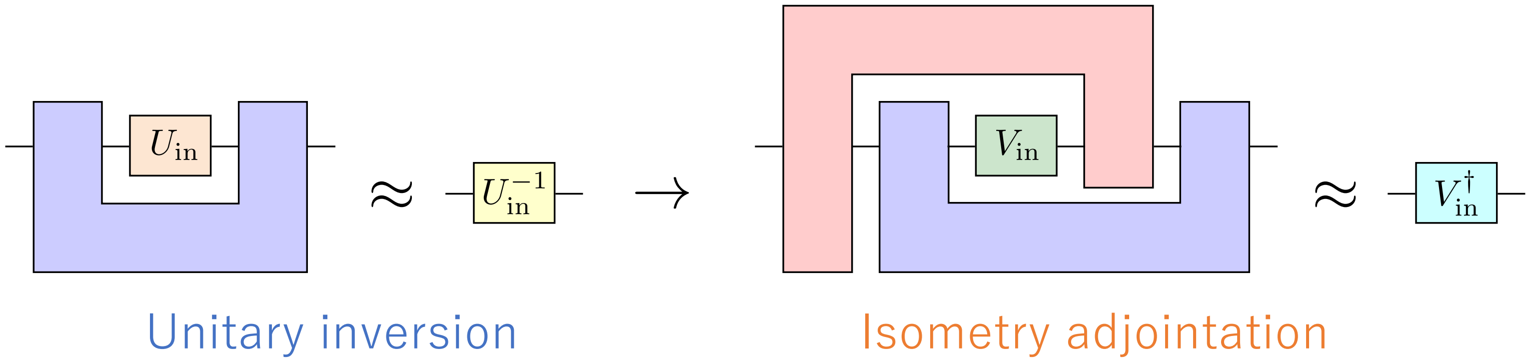


Isometry adjointation

- Result:

SY, A. Soeda and M. Murao, arXiv:2401.10137.

Given unitary inversion protocol, we can construct isometry adjointation protocol.



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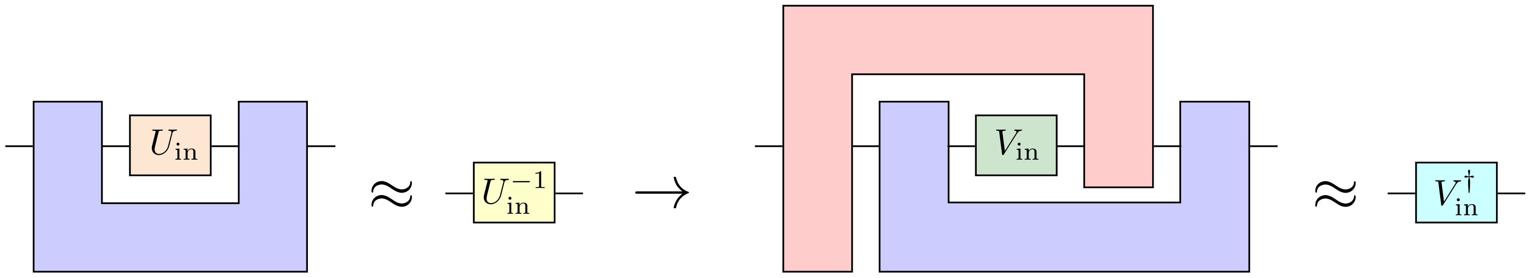


Unitary inversion   Isometry adjointation

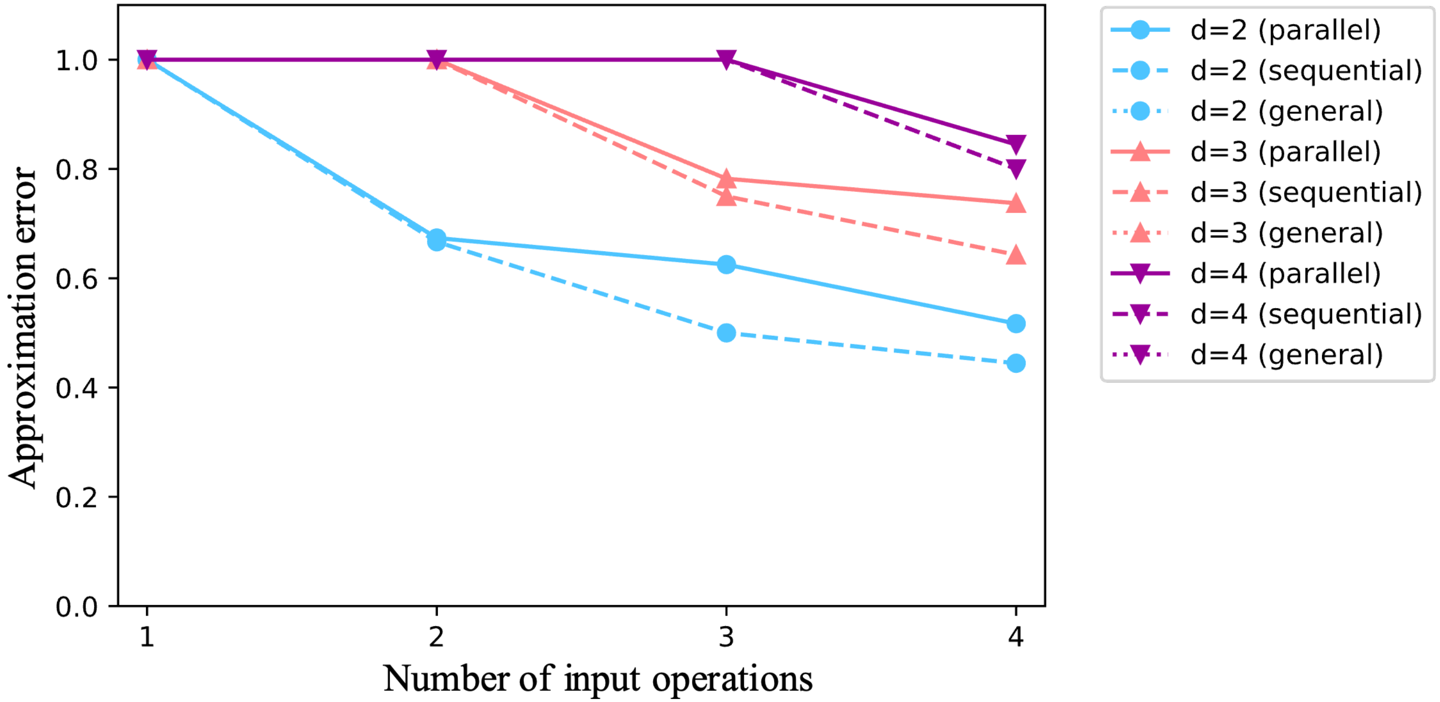
- Achieves the optimal diamond-norm approximation error!
- Optimal approximation error for parallel: $\epsilon = \Theta(d^2/n)$

d: input dimension of isometry, n: number of calls of the input isometry

Numerical results

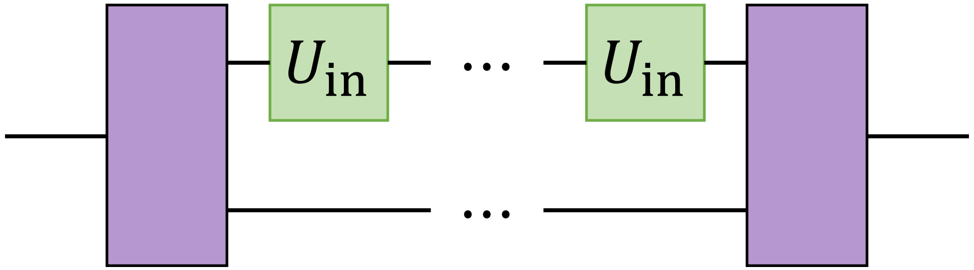
- Minimum value of approximation error

→ Semidefinite programming



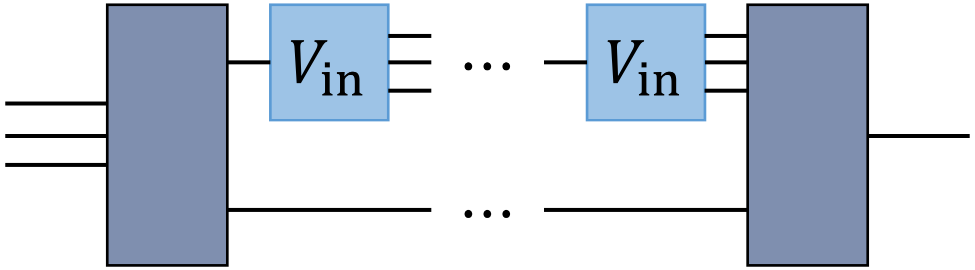
Proof sketch

- Key idea



Unitary inversion

$\mapsto$



Isometry adjointation

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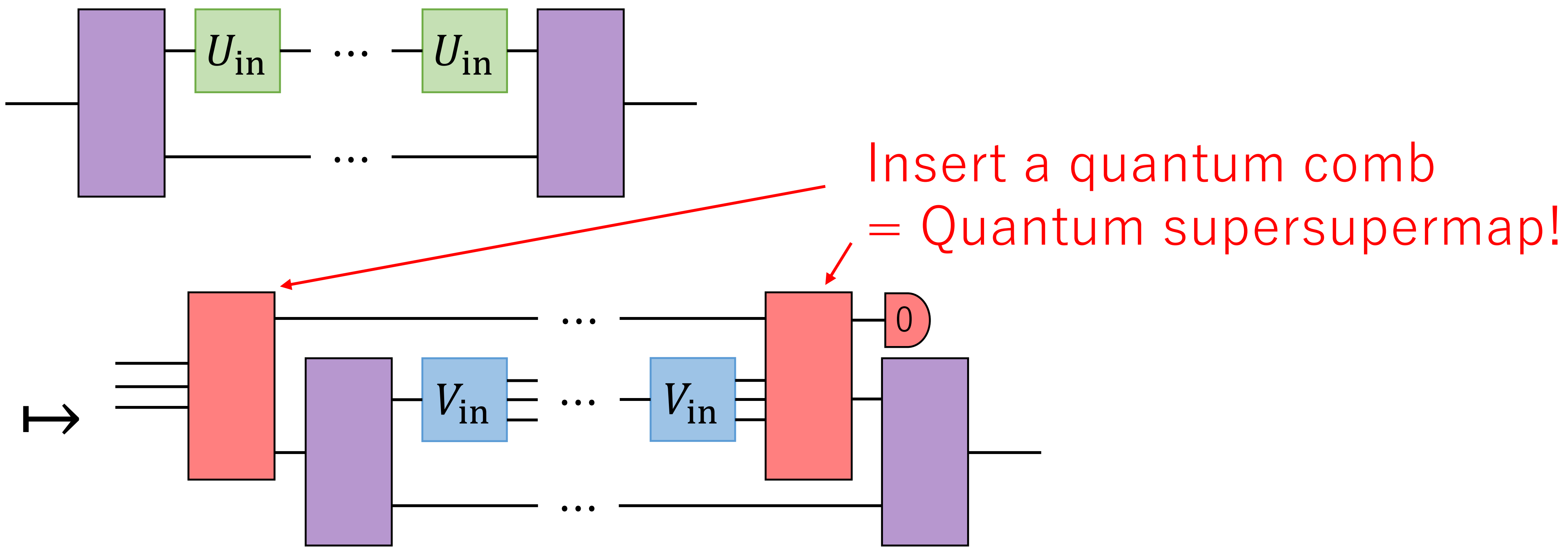

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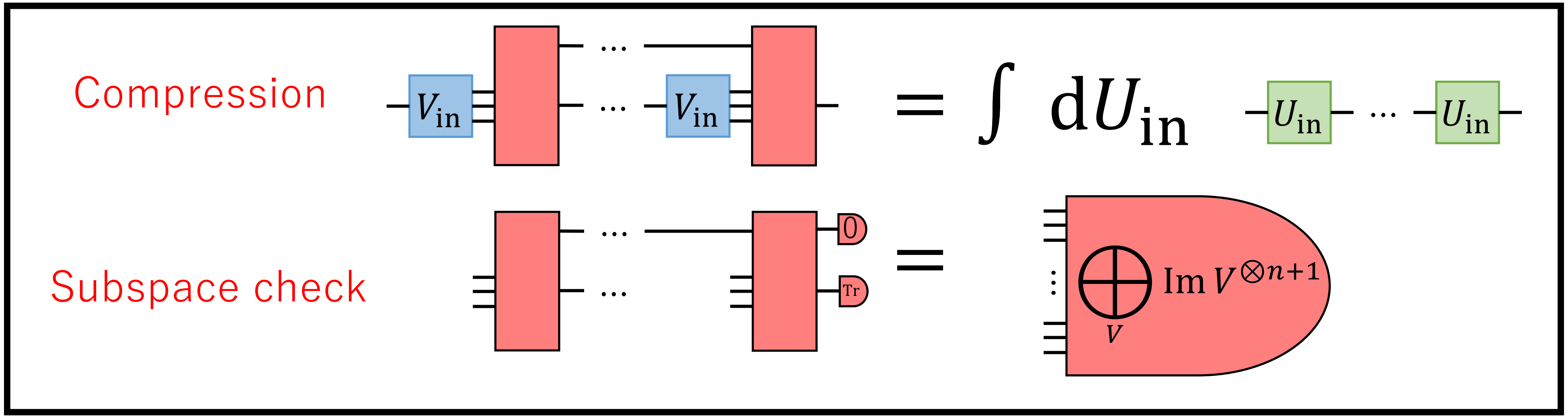

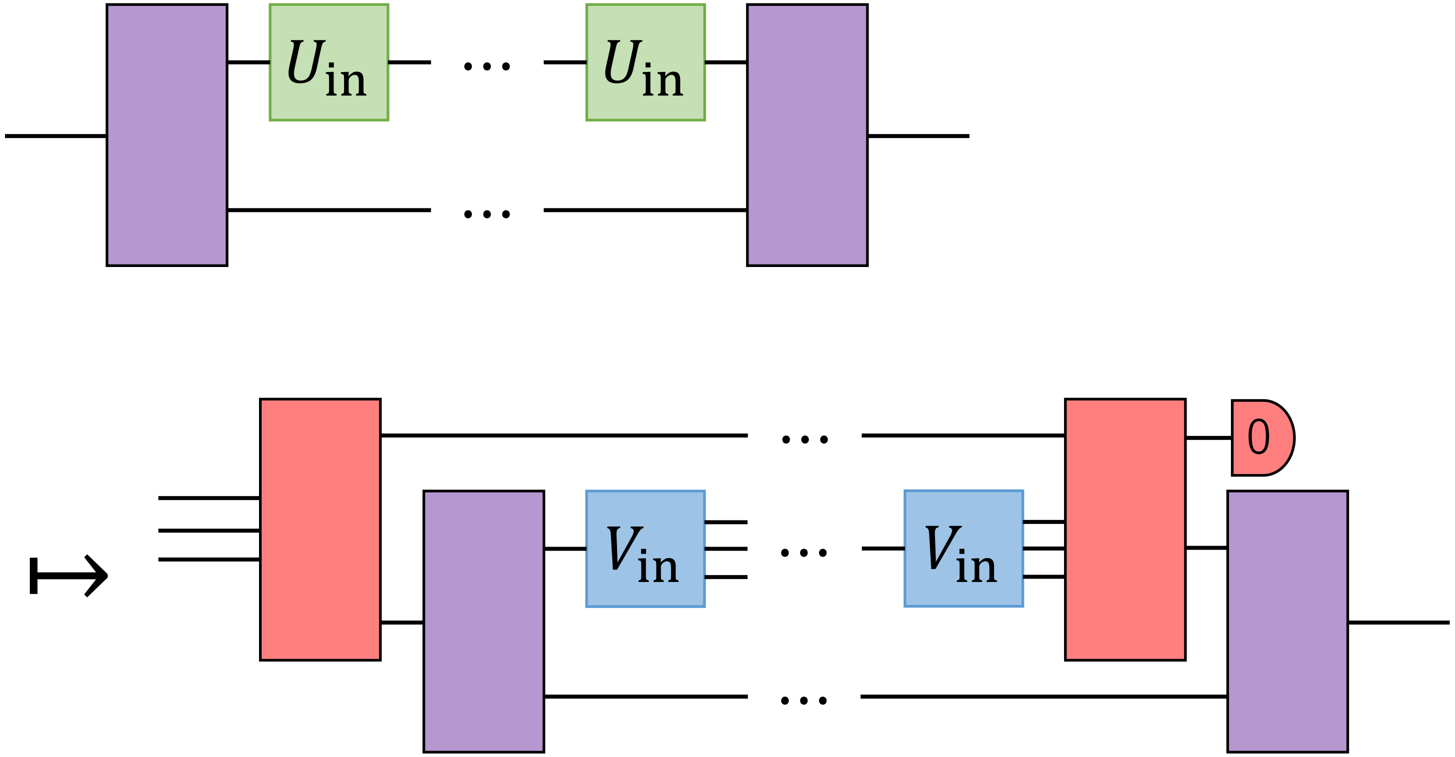

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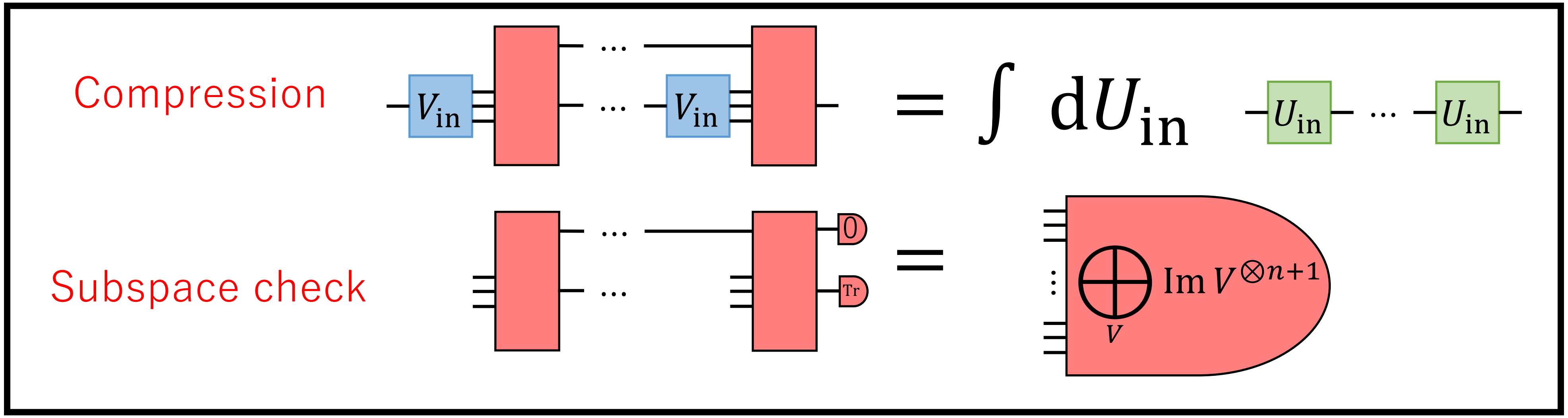

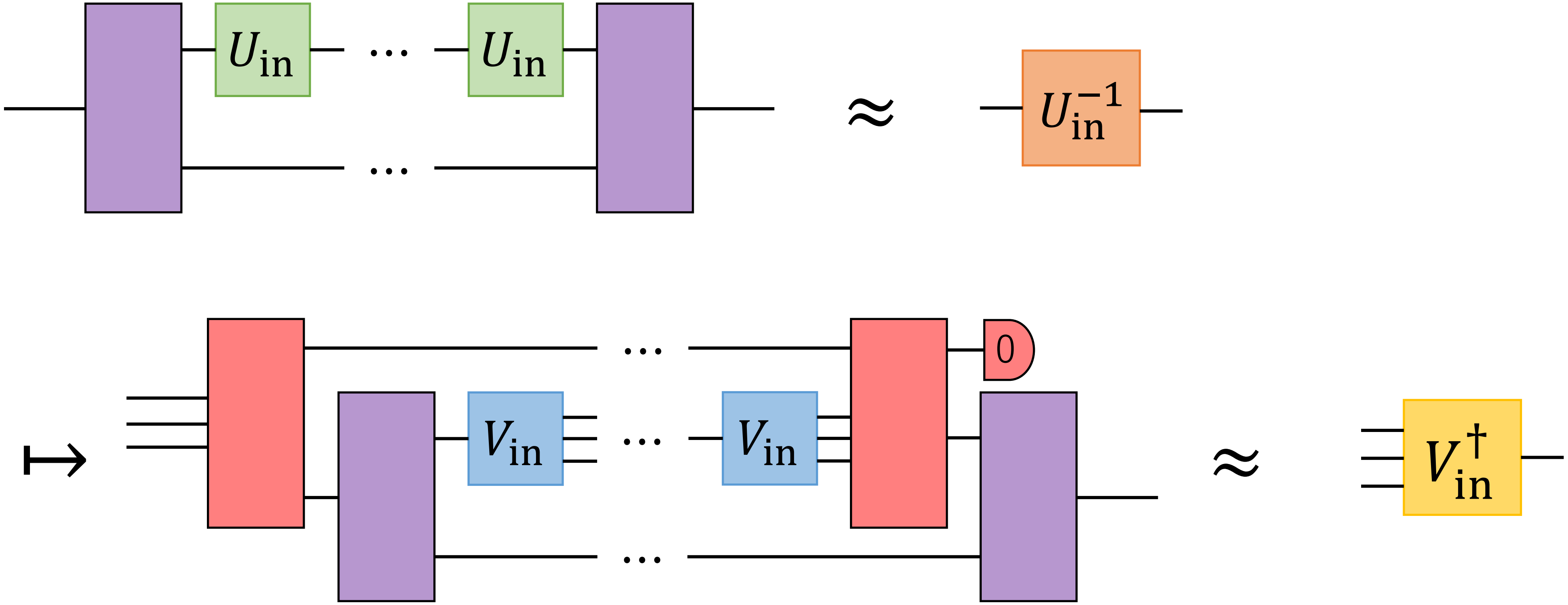

Related tasks

- Isometry inversion: $V_{\mathrm{in}} \mapsto V_{\mathrm{in}}^{-1}$

- Universal error detection: $V_{\mathrm{in}} \mapsto \mathrm{POVM}\ \{\Pi_{\mathrm{Im}\,V_{\mathrm{in}}}, I - \Pi_{\mathrm{Im}\,V_{\mathrm{in}}}\}$

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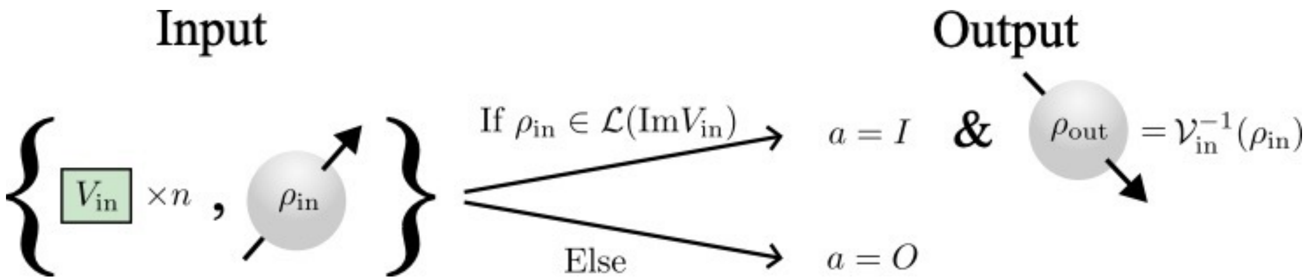

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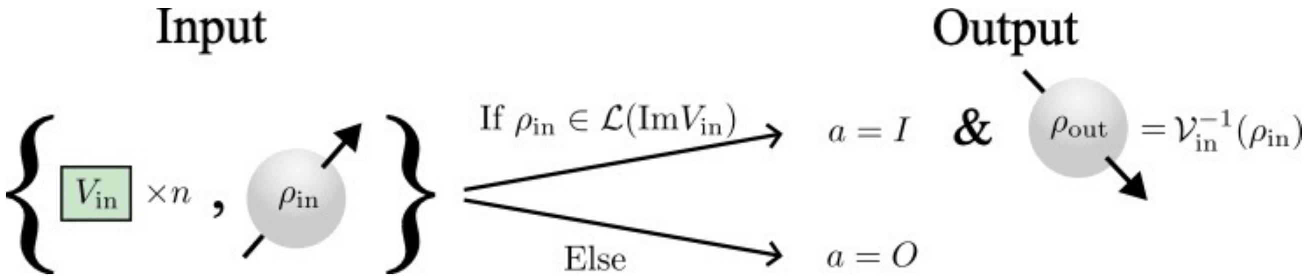

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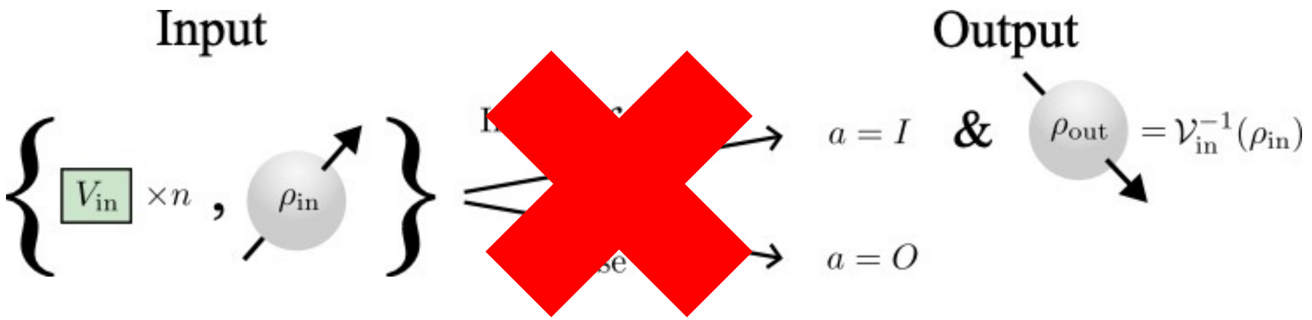

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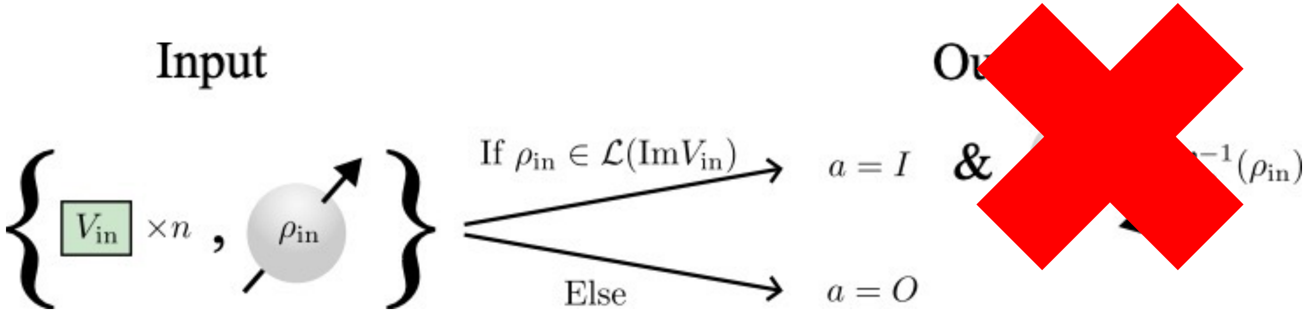

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	Isometry inversion		Unitary inversion	Parallel or sequential protocol
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Optimal approximation error

$n = kd + l\ (k \in \mathbb{Z}, 0 \leq l \leq d)$

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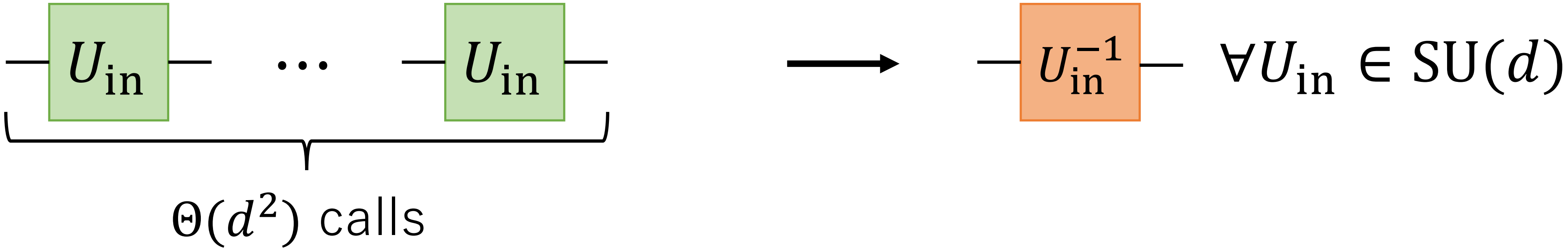
Deterministic and exact isometry inversion

- There exists a deterministic and exact unitary inversion protocol.



SY, Akihito Soeda and Mio Murao, PRL 131, 120602 (2023).
Y.-A. Chen, Y. Mo, Y. Liu, L. Zhang and X. Wang, arXiv:2403.04704.
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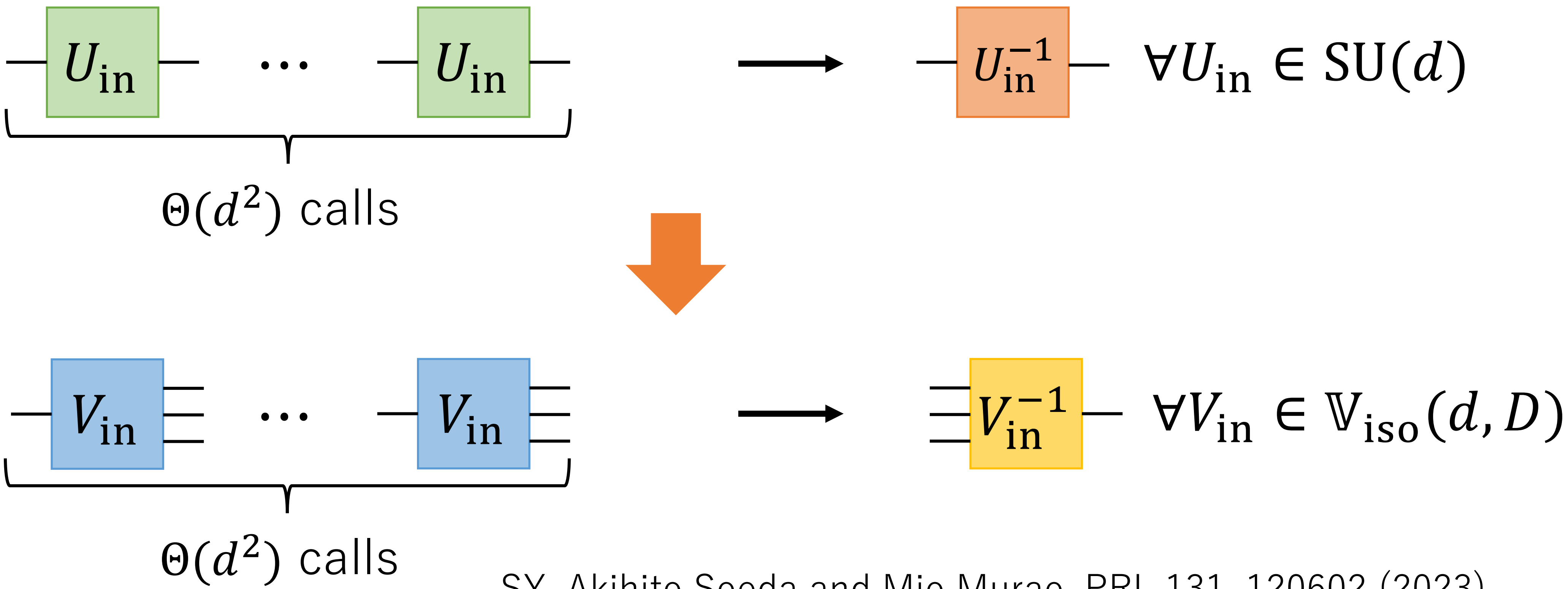

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- Result: Optimal construction of isometry adjointation

- Future works

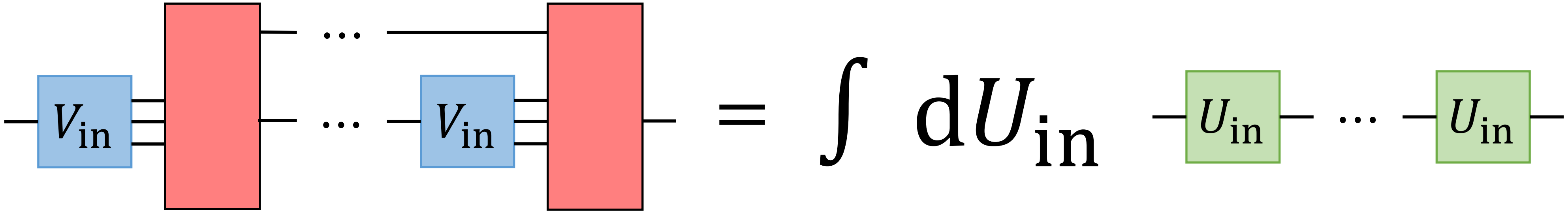
Future works

- Application of isometry adjointation

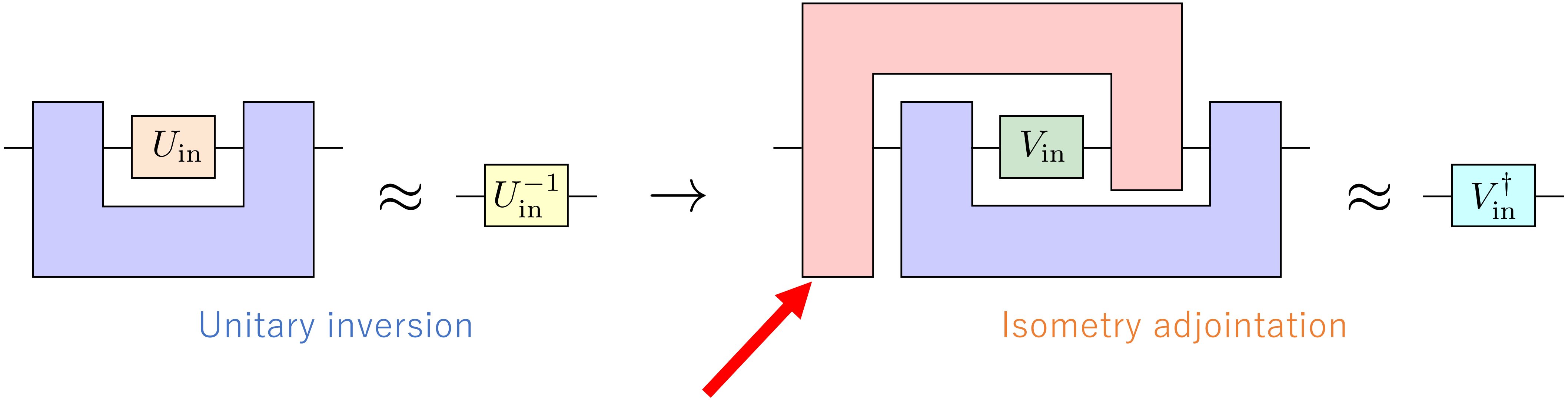
- Property testing?

- Application of the quantum supersupermap

- Compression of the temporal quantum data?



Summary



Inserting another quantum comb!

- Optimal approximation error for parallel: $\epsilon = \Theta(d^2/n)$
- Numerical results based on SDP

arXiv:2401.10137